ON THE PREFRACTURE ZONE MODEL IN ELASTIC BODY AT THE CRACK TIP ON THE INTERFACE OF MEDIA

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ABSTRACT

The plane symmetrical problem on calculation of the prefracture zone at the tip of a crack reaching the interface of isotropic elastic media is considered. The prefracture zone is modelled by lines of rupture of normal displacement located on the interface. An exact solution of the corresponding static problem of the theory of elasticity for piece-homogeneous plane with the media-separating boundary in the form of the sides of angle which contains a semi-infinite crack and two straight lines of rupture emerging from the corner point is constructed by the Wiener-Hopf method. The length of the prefracture zone is determined on the base of this solution. The stress near the corner point is investigated. It is shown that in definite intervals of parameter variation the corner point is the singular point of the above problem of the theory of elasticity. It represents the stress concentrator itself. The exponent of singularity of stresses depends on the angle, Young‘s modulus ratio and on the Poisson‘s ratios. Dependence of the exponent of singularity of stresses on the angle for various intervals of Young‘s modulus ratio variation has been investigated. Parameter variation intervals where the corner point is not the stress concentrator have been determined.

Key words: Displacement rupture line, interface reaching crack, prefracture zone, theory of elasticity, Wiener-Hopf method.

1. INTRODUCTION

There is a lot of specific literature aimed at plane problems of prefracture zone development near the crack tip in a piece-homogeneous body located on the interface of two different media [1-5]. Prefracture zones are modelled by displacement rupture lines emerging from the crack tips. Considerable interest in connection with its possible usage when the problem of braking of the crack in composite materials are solutions of such problems in an other case of mutual position of the interface of media and the crack – in that case when on that interface there is only the crack tip (the crack reaching the interface of media).

For isotropic elastoplastic body plane symmetric problems on the calculation of the prefracture zone at the crack tip, reaching the rough interface of media, in the model frame with a displacement rupture lines have been solved in [6]. Below is given the solution of an analogical problem for the elastic body under the condition that prefracture zone is modelled by a normal displacement rupture lines located on the interface of media.

2. FORMULATION OF THE PROBLEM

Let a piece-homogeneous isotropic elastic body being under the conditions of plane strain, be composed from different homogeneous parts connected in between themselves by a thin connecting layer, the material of which is more brittle than the materials of the given parts. Let‘s consider that the straight crack reaches the rough interface of two media and its tip coincides with the corner point of that interface. The region under consideration is considered to be symmetrical in relation with the straight line, on which the crack is positioned.

Prefracture zone appears and develops with the increase of the outer loading near the crack tip. We will study only the initial stage of its development, considering the outer loading being small enough. Then the size of the prefracture zone will be considerably smaller than the crack length and all the other body sizes.

Taking into account the small portion of the prefracture zone and the gral being to make calculations, we come to the plane static symmetrical problem of the theory of elasticity for the piece-homogeneous isotropic plane with the interface of media in the form of angle sides, having a semi-infinite crack emerging from the vertex when the prefracture zone is available. Asymptotic is being realized at infinity. It is solution of the analogical problem without the prefracture zone, born the smallest at the interval]-1;0[root of its characteristic equation. That problem (problem K) was scrutinized in [6]. Mentioned solution contains arbitrary constant, that is considered to be given. It characterizes the intensity of the external field and must be determined from the solution of the external problem.

In accordance with the hypothesis of localization the initial prefracture zones near the crack tip and other corner points – concentrators of stresses are in themselves thin layers of material – narrow strips emerging from concentrators. Following the hypothesis of localization and taking into consideration properties of the binding material we’ll consider that the initial prefracture zone represents a pair of narrow strips emerging from the crack tip and located on the interface of media.

It’s supposed that in the problem of the theory of elasticity for the finite body which corresponds to the stage of deformation process when the prefracture zone has not appeared yet (problem I), on the interface of media near the crack tip the normal stress is stretching

(condition T). Limitation on the problem parameter enabling the solving of the defined condition, is given below. Due to the fact that the binding material is elastic, main deformations in the prefracture zone are developing on the mechanism of normal rupture. Consequently, the strip-zone will be modelled by the normal displacement rupture line, on which the normal stress is equal to the given constant of the binding material.
Thus, the boundary conditions of the problem of the theory of elasticity which model the process under consideration are as follows (Figure 1):

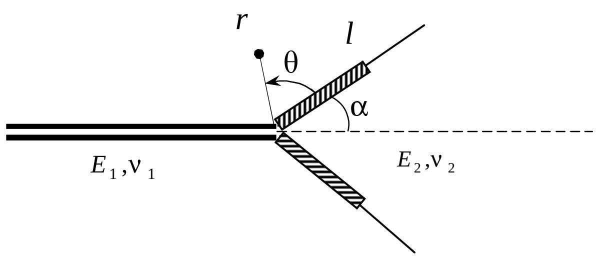

Figure1. Normal displacement ruptures lines at the tip of a crack reaching the interface of media

$$\theta=\pi-\alpha,\ \sigma_\theta=\tau_{r\theta}=0;\ \theta=-\alpha,\ \tau_{r\theta}=0,\ u_\theta=0;$$

$$\theta=0,\ \langle\sigma_\theta\rangle=\langle\tau_{r\theta}\rangle=0,\ \langle u_r\rangle=0; \tag{1}$$

$$\theta=0,\ r<l,\ \sigma_\theta=\sigma;\ \theta=0,\ r>l, \langle u_\theta\rangle=0; \tag{2}$$

$$\theta=0,\ r\to\infty,\ \sigma_\theta=Cgr^{\lambda}+o(1/r), \tag{3}$$

Where $\alpha\le\theta\le\pi-\alpha$; $\langle a\rangle$ is the jump of value a; $\sigma, C>0$, are given constants; λ is the smallest on the interval]-1;0[root of equation

$$\Delta(-x-1)=0, \Delta(z)=b_0(z)+b_1(z)e+b_2(z)e^2, \tag{4}$$

$$b_0(z)=(\sin 2z\alpha+z\sin 2\alpha)\{(1+\chi_1)^2-4[\chi_1\sin^2 z(\pi-\alpha)+z^2\sin^2\alpha]\},$$

$$b_1(z)=(1+\chi_1)(1+\chi_2)\sin 2z\pi+4(\chi_2\sin 2z\alpha-z\sin 2\alpha)[\sin^2 z(\pi-\alpha)-z^2\sin^2\alpha]-$$
$$-(\sin 2z\alpha+z\sin 2\alpha)\{(1+\chi_1)(1+\chi_2)-4[\chi_1\sin^2 z(\pi-\alpha)+z^2\sin^2\alpha]\},$$

$$b_2(z)=-4(\chi_2\sin 2z-z\sin 2\alpha)[\sin^2 z(\pi-\alpha)-z^2\sin^2\alpha]$$

$$\chi_{1,2}=3-4\nu_{1,2},\ e=\frac{1+\nu_2}{1+\nu_1}e_0,\ e_0=\frac{E_1}{E_2},$$

E_1, E_2 are Young's moduli; ν_1,ν_2 are Poisson's ratios; $g(\alpha,e_0,\nu_1,\nu_2)$ is the known function. The values of λ (root of the characteristic equation (4)) for some values of α and e_0 under $\nu_1=\nu_2=0{,}3$ are shown in the Table 1. As shown in the calculation results, function $g(\alpha,e_0,\nu_1,\nu_2)$ under fixed α,ν_1,ν_2 is positive, if

$$0<e_0<e_0^0(\alpha,\nu_1,\nu_2) \tag{5}$$

($e_0^0>1$ is the point where the function is zero).
Considering that e_0 satisfies the (5). Then in the problem I, mentioned above, $\sigma_\theta(r,o)\to+\infty$ if $r\to0$ and the condition T, pointed above, will be fulfilled. Condition T is fulfilled under all α,ν_1,ν_2, if e_0 is less than the smallest value of the function $e_0^0(\alpha,\nu_1,\nu_2)$. In particular,

under $\nu_1 = \nu_2 = 0{,}3$ for the fulfillment of the condition T it is sufficient that e_0 does not exceed three.

Table1. Some values of the root of the characteristic equation

α^0 \ e_0	0,1	0,2	0,3	0,5	2	3	5	10
15	-0,4398	-0,4679	-0,4793	-0,4868	-0,5183	-0,5338	-0,5610	-0,6137
30	-0,4304	-0,4483	-0,4620	-0,4741	-0,5383	-0,5688	-0,6161	-0,6900
45	-0,4252	-0,4352	-0,4476	-0,4620	-0,5588	-0,6020	-0,6614	-0,7404
60	-0,4089	-0,4188	-0,4328	-0,4506	-0,5742	-0,6244	-0,6882	-0,7665
75	-0,3816	-0,3972	-0,4173	-0,4409	-0,5795	-0,6305	-0,6957	-0,7705
90	-0,3467	-0,3757	-0,4066	-0,4370	-0,5737	-0,6201	-0,6792	-0,7546
105	-0,3422	-0,3809	-0,4151	-0,4450	-0,5594	-0,5969	-0,6475	-0,7192
120	-0,4079	-0,4261	-0,4452	-0,4636	-0,5408	-0,5671	-0,6045	-0,6647
135	-0,4620	-0,4679	-0,4748	-0,4824	-0,5227	-0,5381	-0,5603	-0,5991
150	-0,4892	-0,4906	-0,4924	-0,4945	-0,5087	-0,5153	-0,5254	-0,5426
165	-0,4987	-0,4988	-0,4990	-0,4993	-0,5013	-0,5025	-0,5047	-0,5089

Near the tip of the side rupture line taking into account general things about the stress behavior in the vicinity of corner points of elastic body the asymptotic is realized. It represents the solution of a homogeneous problem of the theory of elasticity for piece-homogeneous plane which contains on the straightlined interface of media a semi-infinite normal displacement rupture line, born the root – 1/2 of its characteristic equation. In particular,

$$\theta = 0, r \to l + 0, \sigma_\theta \sim \frac{\chi_1 + e + 1 + \chi_2 e}{2(\chi_1 + e)} \frac{k_I}{\sqrt{2\pi (r - l)}}, \qquad (6)$$

Where k_I is the stress intensity factor at the tip of the normal displacement rupture line, which is to be defined.

The task is to define the length l of rupture lines, which model the prefracture zone, and to investigate the stress near the tip O of the crack.

The length l of the rupture line is defined from the condition of the stress limitation near its tip.

3. SOLUTION OF WINER-HOPF EQUATION AND DEFINING THE STRESS INTENSITY FACTOR

The solution of the formulated problem of the theory of elasticity with boundary conditions (1)-(3) (Figure 1) represents the sum of solutions of the following two problems. The first (problem I) differs from it in the following way: instead of the first condition (2) we have

$$\theta = 0, r < l, \sigma_\theta = \sigma - Cgr^\lambda, \tag{7}$$

and at infinity the stresses decrease as $o(1/r), (\sigma_\theta = o(1/r)$ in (3)).
The second problem is problem K.

Using Mellin's integral transform with a complex parameter p to the equilibrium equations, compatibility equation, Hooke's law, conditions (1) and considering the second condition (2) and condition (7) we come to the following Wiener-Hopf functional equation of problem 1:

$$\Phi^+(p) + \frac{\sigma}{p+1} + \frac{\sigma_1}{p+\lambda+1} = -A\operatorname{tg}p\pi \cdot G(p)\Phi^-(p), \tag{8}$$

$$A = \frac{(1+\chi_1)[1+\chi_1+(1+\chi_2)e]}{2[\chi_1+(1+\chi_1\chi_2)e+\chi_2 e^2]}, \quad G(p) = \frac{G_1(p)}{G_2(p)}$$

$$G_1(p) = 2[\chi_1+(1+\chi_1\chi_2)e+\chi_2 e^2][a_0(p)+a_1(p)e]\cos p\pi,$$

$$G_2(p) = [1+\chi_1+(1+\chi_2)e][b_0(p)+b_1(p)e+b_2(p)e^2]\sin p\pi,$$

$$a_0(p) = (1+\chi_1)(\sin 2p\alpha + p\sin 2\alpha)[\sin 2p(\pi-\alpha) + p\sin 2\alpha],$$

$$a_1(p) = 2(1+\chi_2)(\cos 2p\alpha + \cos 2\alpha)[\sin^2 p(\pi-\alpha) - p^2\sin^2\alpha]$$

$$\sigma_1 = -Cgl^\lambda, \quad \hat{O}^+(p) = \int_1^\infty \sigma_\theta(\rho l, 0)\rho^p d\rho, \quad \hat{O}^-(p) = \frac{E_1}{4(1-\nu_1^2)}\int_0^1 \left\langle \frac{\partial u_\theta}{\partial r}\right\rangle\Bigg|_{\substack{r=\rho l\\ \theta=0}} \rho^p d\rho$$

Where - $\varepsilon_1 < \operatorname{Re} p < \varepsilon_2$, $\varepsilon_{1,2}$ are sufficiently small positive numbers.

Simslar equations are solved in [5-7].

Solution of equation (8) is as follows:

$$\hat{O}^+(p) = -\frac{pG^+(p)}{K^+(p)}\{\frac{\sigma}{p+1}[\frac{K^+(p)}{pG^+(p)} + \frac{K^+(-1)}{G^+(-1)}] + \frac{\sigma_1}{p+\lambda+1}[\frac{K^+(p)}{pG^+(p)} + \frac{K^+(-\lambda-1)}{(\lambda+1)G^+(-\lambda-1)}]\} \ (\operatorname{Re} p < 0)$$

$$\hat{O}^-(p) = \frac{K^-(p)G^-(p)}{A}[\frac{\sigma K^+(-1)}{(p+1)G^+(-1)} + \frac{\sigma_1 K^+(-\lambda-1)}{(p+\lambda+1)(\lambda+1)G^+(-\lambda-1)}] (\operatorname{Re} p > 0) \tag{9}$$

$$\exp[\frac{1}{2\pi i}\int_{-i\infty}^{i\infty}\frac{\ln G(z)}{z-p}dz] = \begin{cases} G^+(p), & \operatorname{Re} p < 0 \\ G^-(p), & \operatorname{Re} p > 0 \end{cases}, \quad K^\pm(p) = \frac{\Gamma(1 \mp p)}{\Gamma(\frac{1}{2} \mp p)}$$

($\Gamma(z)$ is gamma function). Using (6),(9) we can find the stresses and stress intensity factor.

Stress intensity factor at the tip of the rupture line is represented by the formula

$$k_I = \frac{2\sqrt{2}(\chi_1 + e)}{1 + \chi_1 + (1 + \chi_2)e}\sqrt{l}\left[\frac{g\Gamma(\lambda+1)}{\Gamma(\lambda+\frac{3}{2})G^+(-\lambda-1)}Cl^{\lambda} - \frac{2}{\sqrt{\pi}G^+(-1)}\sigma\right] \qquad (10)$$

4. RESULTS

Equalling k_I to zero (see (10)), we get the following formula, aimed at defining the length of the prefracture zone:

$$l = L(\frac{C}{\sigma})^{-1/\lambda}, \quad L = [\frac{\sqrt{\pi}\, g\Gamma(\lambda+1)I_1}{2\Gamma(\lambda+3/2)I_2}]^{-1/\lambda}, \qquad (11)$$

$$I_1 = \exp[\frac{1}{\pi}\int_0^{\infty}\frac{\ln G(it)}{t^2+1}dt, \quad I_2 = \exp[\frac{\lambda+1}{\pi}\int_0^{\infty}\frac{\ln G(it)}{t^2+(\lambda+1)^2}dt].$$

Due to the calculation results, if $e_0' \le e_0 \le e_0''$.
$e_0' \approx 0{,}7212, e_0'' \approx 1{,}1452), \nu_1 = \nu_2 = 0{,}3$ function $L(\alpha)$ IN (11) equals decreasing $(L(\pi) = 0)$, and if $e_0 < e_0'$ and $e_0'' < e_0 \le 3$ - has the only extremum – maximum at the point $\alpha_m(e_0)$. İf e_0 is equal to $0{,}1; 0{,}2; 0{,}3; 0{,}5; 1{,}2; 1{,}4; 2; 3$ then α_m^0 is equal to $20{,}2°; 17{,}3°; 15{,}2°; 14{,}7°; 3{,}2°; 10{,}8°$ $17{,}2°; 19{,}4°$ and $L(\alpha_m)$ is equal to $0{,}0341; 0{,}0564; 0{,}0677; 0{,}0785; 0{,}0109; 0{,}0066;$ $0{,}0033;$ $0{,}0016$.
Thus, in case when $(C/\sigma)^{-1/\lambda}$ weakly changes with the changing α and e_0, if $e_0 < e_0'$ and $e_0'' < e_0 \le 3$ the length of the prefracture zone will be the most, if $\alpha = \alpha_m$. İn the above discussed case the length of the prefracture zone increases with a decreasing α, if $e_0' \le e_0 \le e_0''$.

İt is known that part of the prefracture zone located at the crack tip represents the destruction of material region, where the level of stresses is extremely high. The given region differs at its highest maximum level of deformations, resulting in availability of pores and microcracks. İn that connection,particular interest lies in the analysis of stress behaviour near the corner point O, i.e.,in the study of influence of changing the problem parameters on the exponent of stress singularity at the corner point.

With aim of realization the above mentioned investigation, using (9) and Mellin's formula, we define the stresses in the problem 1. İn particular, we get

$$\tau_{r\theta} = \frac{1}{2\pi i}\int_{\gamma}\frac{F(p,\theta)M(p)}{(p+1)(p+\lambda+1)D(p)}r^{-p-1}dp, \qquad (12)$$

$$D(p) = a_0(p) + a_1(p)e, \quad M(p) = \frac{M_1(p)}{\Gamma(1-p)},$$

$$M_1(p) = p[m_1(p+1) + m_2(p+\lambda+1)]\Gamma(1/2-p)G^+(p)l^{p+1},$$

$$m_1 = \frac{g\Gamma(\lambda+1)}{\Gamma(\lambda+3/2)G^+(-\lambda-1)}Cl^{\lambda}, \quad m_2 = -\frac{2}{\sqrt{\pi}G^+(-1)}\sigma,$$

Where - $\alpha < \theta < \pi - \alpha$; $F(p,\theta)$ is the know entire function of p, having zero of first order in the point $p = -1$; γ is the parallel to the imaginary axis line that lies in the strip - $\varepsilon < \operatorname{Re} p < 0$.

Function $D(p)$ has zeroes of firsty order at the points $p = -1$ and $p = -\lambda_1 - 1$, where λ_1 is the only on the interval]-1;0[root of equation $D(-x-1) = 0$. Function $D(p)$ has not other zeroes in the strip $-1 < \operatorname{Re} p < 0$. Thus, in the strip $-1 < \operatorname{Re} p < 0$ the integrand in (12) has three singularities – simple poles at the points $p = -1; -\lambda - 1; -\lambda_1 - 1$.

Using the data about the singular points of the integrand in (12), applying to the integral (12) the residue theorem and adding the solutions of problems 1 and K, we find the principal terms of the expansion of stress $\tau_{r\theta}$ in asymptotic series if $r \to 0$ in the problem of the theory of elasticity under consideration with boundary conditions (1) – (3) (Figure 1).

The below given formula takes its place:

$$\tau_{r\theta} = r^{\lambda_1} f_1(\theta,\alpha,e_0,\nu_1,\nu_2)C_1 + f_2(\theta,\alpha,e_0,\nu_1,\nu_2)\sigma + f(r,\theta,\alpha,e_0,\nu_1,\nu_2,l,\sigma,C), \qquad (13)$$

$$C_1 = \varphi_1(\alpha,e_0,\nu_1,\nu_2)\sigma l^{-\lambda_1} + \varphi_2(\alpha,e_0,\nu_1,\nu_2)Cl^{\lambda-\lambda_1}$$

($f_1, f_2, \varphi_1, \varphi_2$ are the known functions; $f \to 0$ if $r \to 0$; $-1 < \operatorname{Re} p < 0$.

Under the values of the problem parameters, where the equation $D(-x-1) = 0$ on the interval]-1;0[has not root, in (13) addendum containing r^{λ_1} is not available.

The result analysis drives us to the following conclusions. İn definite intervals of changing of the parameters the corner point O represents in itself the singular point of the considered problem of the theory of elasticity. İt itself represents the stress concentrator. The exponent of singularity of stresses $\lambda_1 \in]-1;0[$ depends on the angle α, Young's modulus ratio $e_0 = E_1 / E_2$ and on the Poısson's ratıos ν_1, ν_2.

İf $e_0^{(1)} \le e_0 \le 1 (e_0^{(1)} \approx 0{,}7521)$, function $\lambda_1(\alpha)$ is decreasing $(\lambda_1(0) = 0, \lambda_1(\pi) = -1/2)$ and, thus, increasing the angle α the stress concentration in the region of the destruction of material increases.

Let $e_0^{(2)} \le e_0 \le e_0^{(1)} (e_0^{(2)} \approx 0{,}3924)$. with the angle α increase from zero to $\alpha_{\max}(e_0)$ the stress concentration in the region of destruction of material first increases, then weakens. İn this, the stress concentration will be maximum, if $\alpha = \alpha_{\min}(e_0)$. With the decrease of e_0 the angle $\alpha_{\min}$ and $\lambda_1(\alpha_{\min})$ decrease and angle $\alpha_{\max}$ and $\lambda_1(\alpha_{\max})$ increase. İf $e_0 \to e_0^{(1)}$, then $\alpha_{\min}, \alpha_{\max} \to 0$ $(\alpha_0^0 \approx 41{,}2°)$. When $e_0 \to e_0^{(2)}$, angle $\alpha_{\max}$ approximately equals to $65{,}3°$ and $\lambda_1(\alpha_{\max}) = 0$. İf $e_0 = 1/2$, then $\alpha_{\min}^0 \approx 24{,}7°; \lambda_1(\alpha_{\min}) \approx -0{,}1154$, and $\alpha_{\max}^0 \approx 55{,}3°$; $\lambda_1(\alpha_{\max}) \approx -0{,}0542$. With the increase of angle α from $\alpha_{\max}(e_0)$ to π , the stress concentration in the region og material destruction increases $(\lambda_1(\pi) = -1/2)$.

Suppose $e_0 < e_0^{(2)}$. with the increase of angle α from zero to $\alpha_{\min}(e_0)$ the stress concentration in the destruction region increase, and with its increase from $\alpha_{\min}(e_0)$ to $\alpha_1(e_0)$ descreases

$(\lambda_1(\alpha_1)=0)$. İf $\alpha_1(e_0)<\alpha<\alpha_2(e_0)(\lambda_1(\alpha_2)=0)$ the corner point O is not the concentrator of stresses. With the decrease of e_0 angle α_2 increases and tends to $\pi/2$ if $e_0\to 0$, and angle α_1 decreases and if $e_0\to 0$ tends to the unique root of equation $2(\pi-\alpha)\cos 2\alpha+\sin 2\alpha=0$, approximately equal to $51{,}3°$. Angle $\alpha_{\min}$ and $\lambda_1(\alpha_{\min})$ decrease with the decrease of e_0, besides $\alpha_{\min}\to 0$, and $\lambda_1(\alpha_{\min})\to -1/2$ if $e_0\to 0$. İf e_0 is equal to 0,3; 0,2; 0,1; 0,01; 0,001 then α_1^0 is equal to $61{,}2°;54{,}8°;54{,}5°;51{,}9°;51{,}6°;\alpha_2^0$ is equal to $69{,}1°;84{,}4°;85{,}1°;88{,}5°;89{,}4°;\alpha_{\min}^0$ is equal to $18{,}6°;14{,}8°;10{,}4°;3{,}2°;2{,}3$ and $-\lambda_1(\alpha_{\min})$ is equal to 0,1732;0,2361; 0,3060; 0,4589; 0,4938. with the increase of angle α from $\alpha_2(e_0)$ to π the stress concentrationin the destruction region of material increases $\lambda_1(\pi)=-1/2$.

İf $1<e_0\le e_0^{(3)}(e_0^{(3)}\approx 1{,}3272)$, then with the angle α increase, the stress concentration in the destruction region increases $\lambda_1(0)=-1,\lambda_1(\pi)=-1/2$.

Let $e_0^{(3)}<e_0\le e_0^{(4)}(e_0^{(4)}\approx 1{,}6392)$. With the increase of angle α concentration of ctresses in the destruction region first increases, and then decreases. İf $\alpha=\alpha_{\min}(e_0)$, concentration of ctreses will be the largest. With the increase of e_0 angle $\alpha_{\min}$ and $\lambda_1(\alpha_{\min})$ decreases. İf $e_0\to e_0^{(3)}$, then $\alpha_{\min}\to\pi$.

Suppose $e_0>e_0^4$. İf $0<\alpha<\alpha_3(e_0)(\lambda_1(\alpha_3)=0)$ the corner point O is not the concentrator of stresses. With the increase of e_0 angle α_3 increases and tends to $\pi/4$ if $e_0\to\infty$. With the increase of angle α from $\alpha_3(e_0)$ to $\alpha_{\min}(e_0)$ concentration of stresses in the destruction region of material increases, and with its increase from $\alpha_{\min}(e_0)$ to π - decreases. Angle $\alpha_{\min}$ and $\lambda_1(\alpha_{\min})$ decrease with the increase of e_0; $\alpha_{\min}\to\pi/2$, and $\lambda_1(\alpha_{\min})\to -1$ if $e_0\to\infty$. To the values of e_0, equal to 2 and 3, correspond the values $\alpha_3^0,\alpha_{\min}^0,-\lambda_1(\alpha_{\min})$, equel to $42{,}4°;109{,}6°;0{,}5270$ and $43{,}1°;98{,}5°;0{,}5834$.

İf $\alpha°\le 47°$ then with the increase of e_0 $(0<e_0<\infty)$, concentration of stresses in the region of material desrtuction decreases, and if $\alpha°\ge 49°$ - increases.

5. CONCLUSIONS

Thus, the plane symmetrical problem on calculation of the prefracture zone at the tip of a crack reaching the interface of isotropic elastic media is investigated. The prefracture zone is modelled by lines of rupture of normal displacement. An exact solution of the corresponding problem of the theory of elasticity is constructed. It is shown that in definite intervals of parameter variation the corner point is the singular point of this problem. It represents the stress concentrator itself. The tend of the stresses to infinity corresponding to that part of the prefracture zone located near the crack tip is the region of material destruction where the stress level is extremely high.

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