

FRACTURE OF A THREE LAYER ELASTIC PANEL

Mehmet Tarık ATAY

Niğde University, Art and Sciences Faculty, Mathematics Department, Niğde, Turkey

ABSTRACT

The panel is symmetrical about both x- and y- axes. The central strip (strip1) of width $2h_1$ contains a central transverse crack of width $2a$ on x-axis. The two strips (strip2) contain transverse cracks of width $c-b$ also on x-axis. The panel is subjected to axial loads with uniform intensities p_1 and p_2 in strip1 and strip2, respectively at $y = \pm\infty$. Materials of all strips are assumed to be linearly elastic and isotropic. Due to double symmetry, only one quarter of the problem ($0 \leq x \leq \infty$ and $0 \leq y \leq \infty$) will be considered. The solutions are obtained by using Fourier transforms both in x and y-directions. Summing several solutions is due to the necessity for sufficient number of unknowns in general expressions in order to be able to satisfy all boundary conditions of the problem. Use of remaining boundary conditions leads the formulation to a system of two singular integral equations. These equations are converted to a system of linear algebraic equations which is solved numerically

Keywords: Crack, Fracture , Stress Intensity Factor, Singular Integral Equations

1. INTRODUCTION

Despite the early works on roots of cause of fracture, quantitative relation between fracture stress and flaw size was made obvious by the work of Griffith, which was published in 1920. With his work on the brittle fracture of glass, he applied a stress analysis on an elliptical hole to the unstable propagation of a crack. Because of their singularity nature and their related problems, strip based problems constitute a significant portion of the field of fracture mechanics. In general, these types of problems can be simplified and represented by boundary value problems, which are solved by both analytical and numerical methods. In this manner, although problems concerning the single crack and collinear cracks in a material have been studied before, three layers of infinite panel with collinear cracks on the x-axis symmetric with respect to the y- axis have not been solved yet.

[1] considered the distribution of stress produced in the interior of an elastic solid by the opening of an internal crack under the action of pressure applied to its surface. In

his work, [2] has pointed out that for somewhat brittle tensile fractures in situations such that a generalized plane-stress and a plane-strain analysis is appropriate, the influence of the test configuration, loads, and crack length upon the stresses near an end of the crack may be expressed in terms of two parameters. Also in his work, it is shown that the other parameter, called stress-intensity factor, proportional to the square root of the crack length and the intensity of force tending to cause crack extension. [3] examined the crack extension in a large plate subjected to general plane loading theoretically and experimentally. [4] considered the problem of two bonded dissimilar semi-infinite planes containing cracks along the bond. [5] studied the problem of determining the stress field in an elastic strip of finite width when pressure is applied to the faces of a Griffith crack situated symmetrically within it. Stress intensity factor is computed by obtaining the numerical solution of the Fredholm integral equation. [6] studied the plane strain problem for a bonded medium composed of three different materials. The integral equations for the general problem are obtained, which turn out to be a system of singular integral equations of the second kind. [7] considered the problem of a laminate composite in presence of a crack located normal to the bond lines. Integral transforms technique is used to formulate the problem in terms of a singular integral equation. [8] studied the problem of edge cracks in an infinite strip. The elastostatic plane problem of an infinite strip containing two symmetrically located internal cracks perpendicular to the boundary is formulated in terms of a singular integral equation with the derivative of the crack surface displacement as the density function. [9] studied the plane problem of a cracked elastic surface layer bonded to an elastic half space. The surface layer is assumed to contain a transverse crack whose surface is subjected to uniform compression. The problem is formulated in terms of a singular integral equation, the derivative of the crack surface displacement being the density function. By using appropriate quadrature formulas, the integral equation reduces to a system of linear algebraic equations. [10] considered the elastostatic plane problem of an infinite strip containing two non-symmetrically located collinear cracks perpendicular to the sides. The strip is assumed to be isotropic and subjected to uniaxial tension. General expressions for field quantities are obtained by using the Fourier transform technique [11] solved the problem of the general plane problem for an infinite strip containing multiple cracks perpendicular to its boundaries. The problem is reduced to a system of singular integral equations

2. FORMULATION AND SOLUTION OF THE PROBLEM

2.1 Formulation

Solution for the infinite panel loaded at infinity having cracks with traction-free surfaces is obtained by superposition of the following two problems: (i) an infinite panel loaded at infinity with no cracks (uniform solution), (ii) an infinite panel with cracks whose surfaces are subjected to the negative of the stresses at the location of these cracks obtained from problem (i) (perturbation problem). These solutions are obtained by using Fourier transforms both in x- and y-directions. Summing several solutions is due to the necessity for sufficient number of unknowns in general expressions in order to be able to satisfy all boundary conditions of the problem. Use of remaining boundary conditions leads the formulation to a system of two singular integral equations. These equations are converted to a system of linear algebraic equations, which is solved numerically.

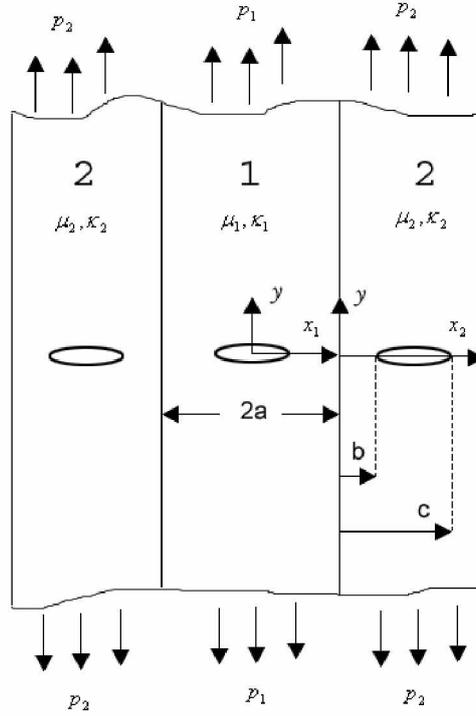


Figure1. Three Layer Panel

2.2 Solution

For linearly elastic, isotropic and two dimensional problems, the field equations can be listed as follows:

Navier Equations:

$$(\kappa + 1) \frac{\partial^2 u}{\partial x^2} + (\kappa - 1) \frac{\partial^2 u}{\partial y^2} + 2 \frac{\partial^2 v}{\partial x \partial y} = 0 \quad \text{and} \quad 2 \frac{\partial^2 u}{\partial x \partial y} + (\kappa - 1) \frac{\partial^2 v}{\partial x^2} + (\kappa + 1) \frac{\partial^2 v}{\partial y^2} = 0 \quad (1.a-b)$$

where u and v are the x - and y - components of the displacement vector; ν being the Poisson's ratio . Stress-Displacement Relations:

$$\frac{1}{2\mu} \tau_{xy} = \frac{1}{2} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \quad \frac{1}{2\mu} \sigma_x = \frac{\kappa + 1}{2(\kappa - 1)} \frac{\partial u}{\partial x} + \frac{3 - \kappa}{2(\kappa - 1)} \frac{\partial v}{\partial y} \quad \text{and} \quad \frac{1}{2\mu} \sigma_y = \frac{3 - \kappa}{2(\kappa - 1)} \frac{\partial u}{\partial x} + \frac{\kappa + 1}{2(\kappa - 1)} \frac{\partial v}{\partial y} \quad (2 a-c)$$

where σ and τ denote normal and shearing stresses, μ is the shear modulus. These equations must be solved with boundary conditions given below:

$$\sigma_{x2}(h_2, y) = 0 \quad (0 \leq y < \infty)$$

$$\begin{aligned}
 \tau_{xy2}(h_2, y) &= 0 & (0 \leq y < \infty) \\
 u_1(h_1, y) &= u_2(0, y) & (0 \leq y < \infty) \\
 v_1(h_1, y) &= v_2(0, y) & (0 \leq y < \infty) \\
 \sigma_{x1}(h_1, y) &= \sigma_{x2}(0, y) & (0 \leq y < \infty) \\
 \tau_{xy1}(h_1, y) &= \tau_{xy2}(0, y) & (0 \leq y < \infty) \\
 \sigma_{y1}(x_1, \infty) &= p_1 & (0 \leq x_1 \leq h_1) \\
 \sigma_{y2}(x_2, \infty) &= p_2 & (0 \leq x_2 \leq h_2) \\
 \sigma_{y1}(x_1, 0) &= 0 & (0 \leq x_1 \leq a) \\
 \sigma_{y2}(x_2, 0) &= 0 & (b < x_2 < c) \\
 v_1(x_1, 0) &= 0 & (a < x_1 < h_1) \\
 v_2(x_2, 0) &= 0 & (0 < x_2 < b \text{ and } c < x_2 < h_2)
 \end{aligned} \tag{3. a-1}$$

in which the subscripts 1 and 2 indicate the central strip (Strip 1) and the strips on the sides (Strip 2), respectively. After applying the boundary conditions to Navier Equations and Stress-Displacement Relations, following integral equations can be obtained.

2.2.1. Integral Equations

The following singular integral equation are obtained

$$\frac{1}{\pi} \int_{-a}^a \frac{4\mu_1 m_1(t)}{(\kappa_1 + 1)} \left[\frac{1}{(t-x)} + k_{11}(x, t) \right] dt + \frac{1}{\pi} \int_b^c \frac{\mu_1 m_2(t)}{(\kappa_2 + 1) p_1} [k_{12}(x, t)] dt = -p_1 \quad (-a < x < a) \tag{4a}$$

$$\frac{1}{\pi} \int_{-a}^a \frac{\mu_2 m_1(t)}{(\kappa_1 + 1)} [k_{21}(x, t)] dt + \frac{1}{\pi} \int_b^c \frac{\mu_2 m_2(t)}{(\kappa_2 + 1) p_2} \left[\frac{4}{t-x} + \frac{4}{t+x} + k_{22}(x, t) \right] dt = -p_2 \quad (b < x < c) \tag{4b}$$

$$k_{ij}(x, t) = \int_0^{\infty} K_{ij}(x, t, \beta) d\beta, \quad (i, j=1, 2) \tag{5}$$

and $K_{ij}(x, t, \beta)$; $(i, j = 1, 2)$ are given in Appendix C. These singular integral equations, Eqs.(2.18a-b), must be solved in such a way that the single-valuedness conditions for the cracks are also satisfied.

$$\int_b^c m_2(t) dt = 0 \quad \text{and} \quad \int_{-a}^a m_1(t) dt = 0 \tag{6 a, b}$$

When the cracks are embedded, kernels $k_{ij}(x, t)$, $(i, j=1, 2)$ are all bounded and there is the simple Cauchy Kernel alone. However, when either (or both) crack touches the interface, $k_{ij}(x, t)$, $(i, j=1, 2)$ contain unbounded parts due to behavior of $K_{ij}(x, t, \beta)$ $(i, j=1, 2)$ as $\beta \rightarrow \infty$. These unbounded parts together with the simple Cauchy Kernel $(t-x)^{-1}$ constitute a set of generalized Cauchy Kernels. After separating bounded and unbounded parts of the integrands, the singular integral equation (4a,b) can be written in the form as follows,

$$\begin{aligned} & \frac{4}{\pi} \int_{-a}^a \left[\frac{1}{t-x_1} + k_{11s}(x_1, t) + \int_0^\infty K_{11b}(x_1, t, \beta) d\beta \right] m_1(t) dt \\ & + \frac{\kappa_1 + 1}{\kappa_2 + 1} \frac{1}{\pi} \int_b^c \left[k_{12s}(x_1, t) + \int_0^\infty K_{12b}(x_1, t, \beta) d\beta \right] m_2(t) dt = -\frac{\kappa_1 + 1}{\mu_1} P_1 \end{aligned} \quad (-a < x_1 < a) \quad (7 a)$$

$$\begin{aligned} & \frac{\kappa_2 + 1}{\kappa_1 + 1} \frac{1}{\pi} \int_{-a}^a \left[k_{21s}(x_2, t) + \int_0^\infty K_{21b}(x_2, t, \beta) d\beta \right] m_1(t) dt \\ & + \frac{1}{\pi} \int_b^c \left[\frac{4}{t-x_2} + \frac{4}{t+x_2} + k_{22s}(x_2, t) + \int_0^\infty K_{22b}(x_2, t, \beta) d\beta \right] m_2(t) dt = -\frac{\kappa_2 + 1}{\mu_2} p_2 \end{aligned} \quad (b < x_2 < c) \quad (7 b)$$

The unknown function $m_1(t)$ is singular at $x_1 = -a$ and $m_2(t)$ is singular at $x_2 = b$ and $x_2 = c$. Their singular behavior can be examined and determined by the complex function technique given in [12]. Their singular behavior can be investigated by first writing;

$$\begin{aligned} m_1(x_1) &= \frac{m_1^*(x_1)}{(a^2 - x_1^2)^\alpha}, \quad -a \leq x_1 \leq a, \quad 0 < \text{Re}(\alpha) < 1 \\ m_2(x_2) &= \frac{m_2^*(x_2)}{(c-x_2)^\gamma (x_2-b)^\Psi}, \quad b \leq x_2 \leq c, \quad 0 < \text{Re}(\gamma, \Psi) < 1 \end{aligned} \quad (8 a,b)$$

where $m_1^*(t)$ and $m_2^*(t)$ are Hölder-continuous functions in the respective intervals $[-a, a]$ and $[b, c]$ and α, Ψ and γ are unknown constants.

2.2.2. The Case Of Embedded Cracks ($a < h_1, 0 < b, c < h_2$)

Calculating the integrals containing simple Cauchy kernels by the formulas given in [12],

$$\begin{aligned} \frac{1}{\pi} \int_{-a}^a \frac{m_1(t) dt}{(t-x_1)} &= \frac{m_1^*(-a) \cot(\pi\alpha)}{(2a)^\alpha (a+x_1)^\alpha} - \frac{m_1^*(a) \cot(\pi\alpha)}{(2a)^\alpha (a-x_1)^\alpha} + F_1(x_1), \\ \frac{1}{\pi} \int_{-a}^a \frac{m_2(t) dt}{(t-x_2)} &= \frac{m_2^*(b) \cot(\pi\Psi)}{(c-b)^\gamma (x_2-b)^\Psi} - \frac{m_2^*(c) \cot(\pi\gamma)}{(c-b)^\Psi (c-x_2)^\gamma} + F_2(x_2) \end{aligned} \quad (9 a,b)$$

are obtained with $\alpha = \Psi = \gamma = 1/2$ by using $\cot(\pi\alpha) = 0, \cot(\pi\Psi) = 0, \cot(\pi\gamma) = 0$ equations. Solutions are in exact agreement with those given in previous works, e.g., [4, 8-10].

2.2.3. Stress Intensity Factors

Stresses become infinite at the edges of the cracks ($x_1 = \pm a, x_2 = b, c; y = 0$). Consequently, the stress state near the crack edges is conveniently expressed by means of the so called stress intensity factors, [13] here, Mode-I (opening mode) stress intensity factors will be given only.

Embedded Crack in Strip1 ($a < h_1$);

$$m_1(x_1) = \frac{m_1^*(x_1)}{\sqrt{a^2 - x_1^2}} = \frac{m_1^*(x_1)(a + x_1)^{-1/2}}{(a - x_1)^{1/2}}, \quad (x_1 \rightarrow a_1), \quad (10)$$

Embedded Cracks in Strip2 ($0 < b, c < h_2$)

$$m_2(x_2) = \frac{m_2^*(x_2)}{\sqrt{x_2 - b}\sqrt{c - x_2}} = \frac{m_2^*(x_2)(c - x_2)^{-1/2}}{(x_2 - b)^{1/2}}, \quad (x_2 \rightarrow b), \quad (11 \text{ 6a,b})$$

$$= \frac{m_2^*(x_2)(x_2 - b)^{-1/2}}{(c - x_2)^{1/2}}, \quad (x_2 \rightarrow c)$$

The Mode-I stress intensity factors are defined in the form

$$K_a = \lim_{x_1 \rightarrow a} \sqrt{2(x_1 - a)} \sigma_{1y}(x_1, 0), \quad (a < h_1)$$

$$K_b = \lim_{x_2 \rightarrow b} \sqrt{2(b - x_2)} \sigma_{2y}(x_2, 0), \quad (0 < b)$$

$$K_c = \lim_{x_2 \rightarrow c} \sqrt{2(x_2 - c)} \sigma_{2y}(x_2, 0), \quad (c < h_2)$$
(12a-c)

Expressions for the necessary stress components at $y = 0$ can conveniently be expressed as

$$\sigma_{1y}(x_1, 0) = \frac{4\mu_1}{1 + \kappa_1} \frac{1}{\pi} \int_{-a}^a \frac{m_1(t)}{t - x_1} dt + \frac{2\mu_1}{(\lambda + \kappa_1)(1 + \lambda\kappa_2)} \frac{1}{\pi} \int_b^c \{ [3 - \kappa_1 - (1 - 3\kappa_2)\lambda]$$

$$+ 2[1 - \kappa_1 - (1 - \kappa_2)\lambda] \frac{x_1 - h_1}{t - x_1 + h_1} \} \frac{m_2(t) dt}{t - x_1 + h_1} + \sigma_{1yb}(x_1, 0),$$
(13 a, b)

$$\sigma_{2y}(x_2, 0) = \frac{2\mu_1}{(\lambda + \kappa_1)(1 + \lambda\kappa_2)} \frac{1}{\pi} \int_{-a}^a \{ [3\kappa_1 - 1 + (1 - \kappa_2)\lambda]$$

$$+ 2[\kappa_1 - 1 + (1 - \kappa_2)\lambda] \frac{x_2}{t - h_1 - x_2} \left. \vphantom{\frac{x_2}{t - h_1 - x_2}} \right\} \frac{m_1(t)dt}{t - h_1 - x_2} + \frac{4\mu_2}{1 + \kappa_2} \frac{1}{\pi} \int_b^c \frac{m_2(t)dt}{t - x_2} + \sigma_{2yb}(x_2, 0)$$

where σ_{1yb} and σ_{2yb} contain all bounded terms.

The singular integrals in these expressions are calculated by using the formulas given in [12]. When $a < h_1$, near the edge at $x_1 = a$:

$$\frac{1}{\pi} \int_{-a}^a \frac{m_1(t)}{t - x_1} dt = -\frac{m_1^*(a)}{\sqrt{2a}} \frac{1}{\sqrt{x_1 a}} + \phi_1(x_1), \quad (14)$$

when $0 < b, c < h_2$, near the edges at $x_2 = b, c$:

$$\frac{1}{\pi} \int_b^c \frac{m_2(t)}{t - x_2} dt = \frac{im_2^*(b)}{\sqrt{c-b}} \frac{1}{\sqrt{x_2 - b}} - \frac{m_2^*(c)}{\sqrt{c-b}} \frac{1}{\sqrt{x_2 - c}} + \phi_2(x_2), \quad (15)$$

Using the appropriate formulas from Eqs.(14)-(15) and also Eqs.(13), the expressions for the stress intensity factors are obtained in the form:

$$K_a = -\frac{4\mu_1}{1 + \kappa_1} \frac{m_1^*(a)}{\sqrt{a}}, \quad (a < h_1), \quad (16)$$

$$K_b = \frac{4\mu_2}{1 + \kappa_2} \frac{m_2^*(b)}{\sqrt{\frac{c-b}{2}}} \quad (0 < b, c < h_2), \quad (17)$$

$$K_c = -\frac{4\mu_2}{1 + \kappa_2} \frac{m_2^*(c)}{\sqrt{\frac{c-b}{2}}}, \quad (0 < b, c < h_2), \quad (18)$$

2.2.4. Numerical Formulation

First, dimensionless variables will be introduced on the cracks.

$$x_1 = a\xi, \quad t = a\eta, \quad (-a < (x_1, t) < a), \quad (19 \text{ a, b})$$

$$x_2 = \frac{c-b}{2}\varepsilon + \frac{c+b}{2}, \quad t = \frac{c-b}{2}\rho + \frac{c+b}{2} \quad (b < (x_2, t) < c) \quad (20 \text{ a, b})$$

so that the singular integral equations, Eqs.(7a,b), and the single valuedness conditions, Eqs.(6a,b), can be rewritten in the form for embedded cracks ($a < h_1, 0 < b, c < h_2$) case, for all integrals in both η and ρ , Gauss-Lobatto quadrature formula [14], will be used. Then, the following linear algebraic equations are obtained:

$$\sum_{i=1}^n C_i \left\{ \left[\frac{1}{\eta_i - \xi_j} + \bar{k}_{11s}(\xi_j, \eta_i) + \bar{k}_{11b}(\xi_j, \eta_i) \right] m_1^{**}(\eta_i) + \left[\bar{k}_{12s}(\xi_j, \rho_i) + \bar{k}_{12b}(\xi_j, \rho_i) \right] m_2^{**}(\rho_i) \right\} = -1$$

$$\sum_{i=1}^n C_i \left\{ \left[\bar{k}_{21s}(\varepsilon_j, \eta_i) + \bar{k}_{21b}(\varepsilon_j, \eta_i) \right] m_1^{**}(\eta_i) + \left[\frac{1}{\rho_i - \varepsilon_j} + \frac{1}{\rho_i + \varepsilon_j + 2\frac{c+b}{c-b}} + \bar{k}_{22s}(\varepsilon_j, \rho_i) + \bar{k}_{22b}(\varepsilon_j, \rho_i) \right] m_2^{**}(\rho_i) \right\} = -1 \quad (j=1, \dots, n-1) \quad (21 \text{ a-c})$$

$$\sum_{i=1}^n C_i m_1^{**}(\eta_i) = 0, \quad \sum_{i=1}^n C_i m_2^{**}(\rho_i) = 0$$

where

$$\begin{aligned} \eta_i &= \rho_i = \cos\left(\frac{i-1}{n-1}\pi\right), \quad (i=1, \dots, n), \\ \xi_j &= \varepsilon_j = \cos\left(\frac{2j-1}{2n-2}\pi\right), \quad (j=1, \dots, n-1), \\ C_i &= \frac{1}{(n-1)}, \quad (i=2, \dots, n-1); \quad C_1 = C_n = \frac{1}{2(n-1)} \end{aligned} \quad (22 \text{ a-c})$$

$m_1^{**}(\eta)$ and $m_2^{**}(\rho)$ are defined by

$$\begin{aligned} \bar{m}_1(\eta) &= \frac{m_1^{**}(\eta)}{\sqrt{1-\eta^2}}, \\ \bar{m}_2(\rho) &= \frac{m_2^{**}(\rho)}{\sqrt{1-\rho^2}}, \end{aligned} \quad (23a, b)$$

3. NUMERICAL RESULTS

3.1 Tabulated Results

Table 3.1 Comparison of the results of this study with the results of [8]

| a/h Gupta(1974) | b/h Gupta(1974) | k _b Gupta(1974) | k _b Present Study | k _c Gupta(1974) | k _c Present Study |
|--------------------|--------------------|-------------------------------|------------------------------------|-------------------------------|------------------------------------|
| 0.1 | 0.5 | 1.1746 | 1.1755 | 1.1169 | 1.1144 |
| 0.2 | 0.6 | 1.1102 | 1.1086 | 1.0961 | 1.0936 |
| 0.4 | 0.8 | 1.0984 | 1.0963 | 1.1250 | 1.1217 |
| 0.5 | 0.9 | 1.1290 | 1.1272 | 1.2278 | 1.2210 |
| 0.6 | 1.0 | 1.6080 | 1.6020 | à ∞ | à ∞ |

3.2 Some Figures

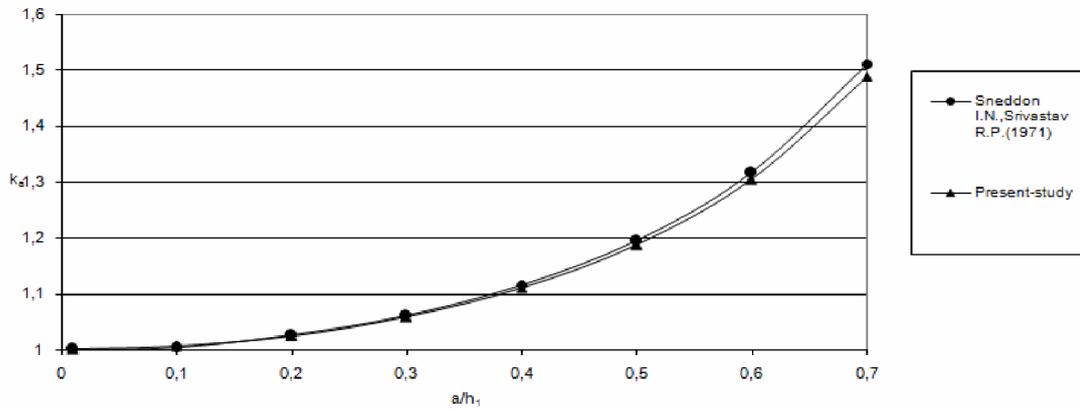


Figure 3.1. Comparison of the Results for an Infinite Strip with a Transverse Central Crack $\mu_1 = \mu_2 = 0.3$ (plane strain) $R1 = 0.4$

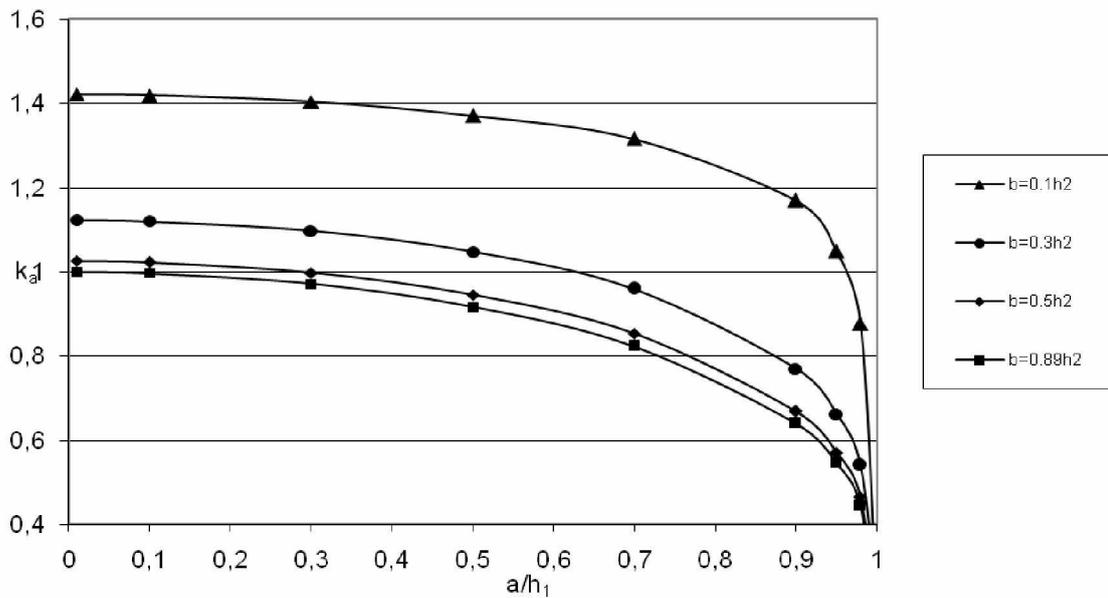


Figure 3.2. Variation of k_a with a/h_1 for 1:epoxy, 2:aluminum, $\mu_2 / \mu_1 = 23.077$, $h_2 = h_1$, $c = 0.9h_2$ (Plane Stress)

4. COCLUSIONS

The values given in Table 3.1 and Fig.3.1 show the results of this study are in good agreement with those given in [8] and [5].

As an example Fig.3.2 show the variation of k_a with a/h_1 for 1:epoxy, 2:aluminum, $\mu_2/\mu_1=23.077$, $h_2=h_1$, $c=0.9h_2$ (Plane Stress) case.

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