THE DELAMINATION BUCKLING OF THE ELASTIC AND VISCOELASTIK RECTANGULAR COMPOSITE PLATE CONTAINING A BAND-CRACK

Surkay D. AKBAROV[a,b], Nazmiye YAHNIOGLU[a] and Ordjali G. RZAYEV[c]

[a] Yildiz Technical University, Faculty of Chemical and Metallurgical Engineering, Department of Mathematical Engineering, Davutpasa Campus, Istanbul Turkey.

[b] Institude of Mathematics and Mechanics of National Academy of Sciences of Azerbaijan, Baku, Azerbaijan

[c] Genje State University, Fac. General Technical Discipline, Dep. Technical Discipline, 374700 Azerbaijan

ABSTRACT

The delamination buckling problem for the rectangular plate made from elastic and viscoelastic composite material is studied. It is supposed that the plate has a band-crack and edge-surfaces of that have an initial infinitesimal imperfection. The development of this initial imperfection with an external compressive loading acting along the crack (for elastic composites) or with time (for a viscoelastic composites) for fixed external loading is studied by the use of the three-dimensional geometrically non-linear field equations of the theory of viscoelasticity for anisotropic bodies. For the determination of the values of the critical forces or of the critical time the initial imperfection criterion is used. The corresponding boundary-value problems are solved by employing boundary form perturbation techniques, Laplace transform and the FEM. The numerical results on the critical force and on the critical time are presented.

Keywords: Band-crack, buckling, critical force, critical time, delamination, rectangular plate, viscoelastic composite.

BANT-ÇATLAK İÇEREN ELASTİK VE VİSKOELASTİK DİKDÖRTGEN PLAĞIN DELAMİNASYON BURKULMASI

ÖZET

Elastik ve viskoelastik kompozit malzemeden yapılmış dikdörtgen plak için delaminasyon burkulma problemi çalışılmıştır. Plağın bant-çatlak içerdiği ve çatlak yüzeylerinin çok küçük eğintiye sahip olduğu kabul edilmektedir. Çatlak doğrultusunda etkiyen dış basınç kuvvetiyle (elastik kompozit için) küçük eğintilerin büyümesi veya sabit dış yükleme altında (viskoelastik kompozit için) zamanla) küçük eğintilerin büyümesi, anizotrop cisim için viscoelastik teorisinin üç boyutlu geometrik lineer olmayan alan denklemleri kullanılarak çalışılmıştır. Kritik kuvvet veya kritik zaman değerini belirlemek için başlangıç eğinti kriteri kullanılmıştır. Uygun sınır değer problemleri sınır tipli pertürbasyon yöntemi, Laplace dönüşümü ve FEM kullanılarak çözülmüştür. Kritik kuvvet veya kritik zamana ait sayısal sonuçlar verilmiştir.

Anahtar kelimeler: Bant-çatlak, burkulma, delaminasyon, dikdörtgen plak, kritik kuvvet, kritik zaman, viskoelastik kompozit.

1. INTRODUCTION

One of the most common failure mechanisms in laminated composite materials is a local buckling around the delaminated zone, i.e. of the zone in which two adjacent layers are partially debounded at their interface. Note that this zone may be formed as a consequence of various impact events, poor fabrication processes and fatique. As is well known, the compressive strength of structures made from undirected fibrous-laminated composite materials may be reduced several times by the presence of this delamination damage, which is modelled as a crack in the related investigations. The review of these investigations is given in the papers by Hwang et al. [1], Short et al. [2], Arman et al. [3] and others in which it is supposed that there is a crack which is parallel to the free plane and to the direction of the compressed external forces. In this case the beginning of the delamination growing process is modelled as a buckling of the part of the material which occupies the region between the crack and the free plane and solutions are found in the framework of the approximate stability loss theories of plates or beams. It is evident that the results of these investigations do not apply in the cases where the thickness of this part is equal or greater than the length of the crack. Moreover, mentioned approximate buckling-delamination theories do not take the singularity of the stresses and strains at the crack tips into account. Therefore it was necessary for the development of the buckling-failure theories in the framework of the Three Dimensional Linearized Theory of Stability (TDLTS) of the deformable solid body mechanics, which has been carried out by Guz [4,5] and by Guz and Nazerenko [6,7] and other. It should be noted that these investigation were made for time-independent composite materials. In the papers by Akbarov [8], Akbarov et al. [9], Akbarov and Yahnioglu [10] the TDLTS was developed for the time-dependent materials. Using this development in the papers Akbarov and Rzayev [11,12,13] and Rzayev and Akbarov [14] the delamination buckling problems for the viscoelastic plate-strip and circular plates were studied. However in these investigations the plane strain-state or axisymmetric strain state were considered.

In the present paper the aforementioned method and investigations are developed for the three-dimensional delamination buckling problem for the rectangular simply supported viscoelastic composite plate containing a band crack.

Throughout the investigations repeated indices are summed over their ranges, however, underlined repeated indices are not summed. Moreover, throughout the investigations, the considered composite material is modelled as an orthotropic, viscoelastic material with normalized mechanical properties. For the solution of the corresponding boundary value problems the boundary form perturbation techniques, Laplace transform and the FEM are employed. The numerical results obtained for the considered three-dimensional problem is compared with those attained for the corresponding problem regarding the plane-strain state.

2. FORMULATION OF THE PROBLEM AND SOLUTION METHOD

Consider a plate which occupies the region (Fig. 1)

$$\Omega = \{0 \le x_1 \le \ell_1,\ 0 \le x_2 \le h,\ 0 \le x_3 \le \ell_3\} \qquad (1)$$

in the Cartesian coordinate system $Ox_1x_2x_3$ and this plate contains a band crack at

$$\{(\ell_1 - \ell_0)/2 < x_1 < (\ell_1 + \ell_0)/2,\ x_2 = h_A,\ 0 \le x_3 \le \ell_3\} \qquad (2)$$

Where ℓ_0 is a length of the crack. It is supposed that the edges of the crack have an insignificant initial imperfection and this imperfection is symmetric with respect to $x_1 = \ell_1/2$ plane. The equations of the crack edge-surfaces can be written as

$$x_2^{\pm} = h_A + \varepsilon f^{\pm}(x_1) \ \text{ for } \ell_1/2 - \ell_0/2 < x_1 < \ell_1/2 + \ell_0/2,\ 0 \le x_3 \le \ell_3 \qquad (3)$$

where ε is a dimensionless small parameter ($\varepsilon << 1$) which characterize the degree of the initial imperfection of the crack edge-surfaces. Moreover, it is supposed that

$$f^{\pm}((\ell_1 - \ell_0)/2) = f^{\pm}((\ell_1 + \ell_0)/2) = 0$$

$$\frac{df^{\pm}((\ell_1 - \ell_0)/2)}{dx_1} = \frac{df^{\pm}((\ell_1 + \ell_0)/2)}{dx_1} = 0 \qquad (4)$$

The geometry of the plate and the crack is illustrated schematically in Fig. 1.

We will consider two mode of the initial imperfection of the crack edge-surfaces. For the first mode we assume that

$$f^{+}(x_1) = f^{-}(x_1) \ \text{(sinphase mode)}, \qquad (5)$$

but for the second mode we assume that

$$f^{+}(x_1) = -f^{-}(x_1) \ \text{(antiphase mode)}. \qquad (6)$$

We assume that the plate material is orthotropic viscoelastic.

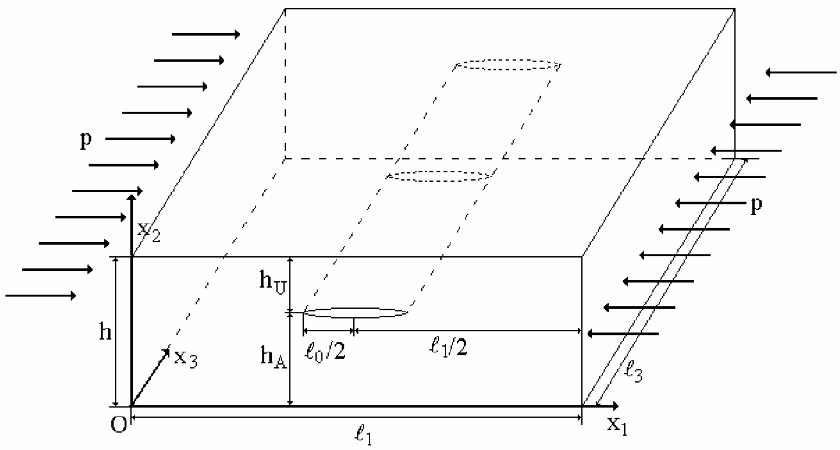

Fig. 1. The geometry of the considered plate

Within the framework of the above-stated we investigate the development of the foregoing insignificant initial imperfections under compression of the plate along the Ox_1 axis with uniformly distributed normal forces with intensity p. At the same time, we assume that at all ends of the plate the displacement along the plate thickness is equal to zero. Note that the aforementioned development will be studied by the use of the three-dimensional geometrically non-linear equations of the theory of viscoelasticity for the orthotropic body. The governing field equations of this theory are

$$\frac{\partial}{\partial x_j}\left[\sigma_{jn}\left(\delta_i^n + \frac{\partial u_i}{\partial x_n}\right)\right] = 0,$$

$$\sigma_{ij} = C_{ijrs0}\varepsilon_{rs}(t) + \int_0^t C_{ijrs}(t-\tau)\varepsilon_{rs}(\tau)d\tau,$$

$$\varepsilon_{rs} = \frac{1}{2}\left(\frac{\partial u_r}{\partial x_s} + \frac{\partial u_s}{\partial x_r} + \frac{\partial u_n}{\partial x_s}\frac{\partial u_n}{\partial x_r}\right),\ i;j;r;s=1,2,3. \tag{7}$$

In (7) the conventional notation is used.

The following boundary conditions are satisfied.

$$u_2\big|_{x_1=0;\ell_1} = 0,\ u_2\big|_{x_3=0;\ell_3} = 0,\ \left[\sigma_{1n}\left(\delta_1^n + \frac{\partial u_1}{\partial x_n}\right)\right]\Bigg|_{x_1=0;\ell_1} = p,$$

$$\left[\sigma_{3n}\left(\delta_3^n + \frac{\partial u_3}{\partial x_n}\right)\right]\Bigg|_{x_3=0;\ell_3} = 0,\ \left[\sigma_{2n}\left(\delta_i^n + \frac{\partial u_i}{\partial x_n}\right)\right]\Bigg|_{x_1=0;h} = 0,\ i=1,2,3 \tag{8}$$

$$\left[\sigma_{jn}\left(\delta_i^n + \frac{\partial u_i}{\partial x_n}\right)\right]\Bigg|_{\substack{x_2^{\pm}=h_A+\varepsilon f^{\pm}(x_1) \\ (\ell_1-\ell_0)/2<x_1<(\ell_1+\ell_0)/2 \\ 0<x_3<\ell_3}} n_j^{\pm} = 0 \tag{9}$$

where $n_j^{\pm}$ are the ortho-normal components of the unit normal vector of the crack edge-surfaces.

With the above-stated the formulation of the considered problem is exhausted. Now we consider the method of solution of this problem.

First, using the equations $x_2^{\pm} = h_A + \varepsilon f^{\pm}(x_1)$ we derive the following expressions for $n_j^{\pm}$:

$$n_1^{\pm} = \frac{\pm\varepsilon\dfrac{\partial f^{\pm}(x_1)}{\partial x_1}}{\sqrt{1+\varepsilon^2\left(\dfrac{\partial f^{\pm}(x_1)}{\partial x_1}\right)^2}},\ n_2^{\pm} = \frac{\pm 1}{\sqrt{1+\varepsilon^2\left(\dfrac{\partial f^{\pm}(x_1)}{\partial x_1}\right)^2}},\ n_3^{\pm} = 0 \tag{10}$$

Assume that $\varepsilon\left|\dfrac{\partial f^{\pm}(x_1)}{\partial x_1}\right| << 1$, according to which, we can represent the expression (10) in the series form in the parameter ε:

$$n_1^{\pm} = \sum_{k=0}^{\infty}\varepsilon^{2k+1}n_{1k}^{\pm}(x_1),\ n_2^{\pm} = \sum_{k=0}^{\infty}\varepsilon^{2k}n_{2k}^{\pm}(x_1). \tag{11}$$

For constriction paper the explicit expressions of the coefficients $n_{1k}^{\pm}(x_1)$ and $n_{2k}^{\pm}(x_1)$ are not given here. According to Akbarov [8], Akbarov and Yahnioglu [10], Akbarov et al. [9], Akbarov and Rzayev [11,12,13], Rzayev and Akbarov [14], we represent also the sought values in the series form in the parameter ε:

$$\left\{\sigma_{ij};\varepsilon_{ij};u_i\right\} = \sum_{q=0}^{\infty}\varepsilon^q\left\{\sigma_{ij}^{(q)};\varepsilon_{ij}^{(q)};u_i^{(q)}\right\}. \tag{12}$$

Substituting (12) into (7) and (8) and comparing identical powers of ε to describe each approximation, we attain the corresponding closed system equations and boundary conditions. Owing to the linearity of the mechanical relations in (7) and the first two end conditions in (8), they will be satisfied for each approximation (12) separately. The remaining relations obtained from (7) and (8) for every q-th approximation contains the values of all the previous approximations. At the same time, under satisfying the edge-surface conditions (9) we expand the values of each approximation (12) in series in the vicinity of $(x_1, h_A \pm 0, x_3)$ and using the expressions (11) the corresponding conditions are attained from (9).

Thus, supplying the unknowns with corresponding indexes in the equations (7) and (8) we obtain the corresponding ones for the zeroth approximation. In this case from (9) we attain that

$$\left[\sigma_{2n}^{(0)}\left(\delta_i^n + \frac{\partial u_i^{(0)}}{\partial x_n}\right)\right]\Bigg|_{\substack{x_2^{\pm}=h_A \pm 0 \\ (a_1-a_0)/2<x_1<(a_1+a_0)/2 \\ 0<x_3<a_3}} = 0. \qquad (13)$$

It follows from the well-known mechanical consideration that for the comparatively rigid composites under determination of the zeroth approximation we can use the relation $\delta_i^n + \frac{\partial u_i^{(0)}}{\partial x_n} \approx \delta_i^n$, according to which, the stresses in the zeroth approximation are determined as follows

$$\sigma_{11}^{(0)} = p, \ \sigma_{ij}^{(0)} = 0 \ \text{for} \ ij \neq 11. \qquad (14)$$

Moreover, according to the relation $\delta_i^n + \partial u_i^{(0)} \big/ \partial x_n \approx \delta_i^n$, we obtain the following equations and boundary conditions for the first approximation.

$$\frac{\partial}{\partial x_j}\left[\sigma_{ji}^{(1)} + \sigma_{jn}^{(0)} \frac{\partial u_i^{(1)}}{\partial x_n}\right] = 0, \ \varepsilon_{ij}^{(1)} = \frac{1}{2}\left(\frac{\partial u_i^{(1)}}{\partial x_j} + \frac{\partial u_j^{(1)}}{\partial x_i}\right) \qquad (15)$$

$$u_2^{(1)}\Big|_{x_1=0;a_1} = 0, \ u_2^{(1)}\Big|_{x_3=0;a_3} = 0, \ \left[\sigma_{11}^{(1)} + \sigma_{11}^{(0)} \frac{\partial u_1^{(1)}}{\partial x_1}\right]\Bigg|_{x_1=0;a_1} = 0,$$

$$\sigma_{33}^{(1)}\Big|_{x_3=0;a_3} = 0, \ \sigma_{2i}^{(1)}\Big|_{x_2=0;h} = 0, \ i=1,2,3 \qquad (16)$$

$$\sigma_{2i}^{(1)}\Big|_{\substack{x_2^{\pm}=h_A \pm 0 \\ (a_1-a_0)/2<x_1<(a_1+a_0)/2 \\ 0<x_3<a_3}} = «\sigma_{11}^{(0)} \frac{\partial f^{\pm}}{\partial x_1} \delta_i^1, \ i=1,2,3. \qquad (17)$$

Note that the equations (15) and the conditions (16), (17) are attained from equations (7) (except the constitutive relations) and the conditions (8), (9) respectively.

Without limiting generality let us assume that the material of the considered plate is homogeneous, orthotropic with the principle axes of elastic symmetries Ox_1, Ox_2 and Ox_3. In this case, the constitutive relations of the plate material can be written as follows.

$$\sigma_{ii}^{(q)} = A_{ij}\varepsilon_{jj}^{(q)}, \ i;j=1,2,3, \ \ \sigma_{12}^{(q)} = 2A_{66}\varepsilon_{12}^{(q)},$$

$$\sigma_{13}^{(q)} = 2A_{55}\varepsilon_{13}^{(q)},\ \sigma_{23}^{(q)} = 2A_{44}\varepsilon_{23}^{(q)},\ q=0,1,2,\ldots \tag{18}$$

In (18) $A_{11},\ldots,A_{66}$ are the following operators:

$$A_{ij}\varphi(t) = A_{ij0}\varphi(t) + \int_0^t A_{ij1}(t-\tau)\varphi(\tau)d\tau,\ ij=11;22;33;12;13;23;44;55;66 \tag{19}$$

The notation used in (18) and (19) is obvious.

By the similar manner we obtain the corresponding equations and boundary conditions for the second and subsequent approximations. Yet, in these cases the equilibrium equations become the inhomogeneous ones and the geometrical relations contain the additional terms of the previous approximations.

Thus, the investigation of the growth of the initial insignificant crack edge-surface imperfections which is contained by the plate with the flow of time under the action of the external compressive forces is reduced to the solutions to the series of boundary-value problems such as (15)-(18). In the papers by Akbarov and Rzayev [11,12,13] and others, by direct verification it is proven that the linear equations (15)-(17) coincide with the corresponding equations TDLTS which are presented by Guz [2,3]. Moreover, the homogeneous parts of the equations and relations regarding the second and subsequent approximations are also coincide with the corresponding ones of TDLTS.

After determination of the stress-deformation state in the plate (using the above-described solution procedure) it is necessary to select the stability loss criteria. According to Hoff [15], as a stability loss criterion we will assume the case where the size of the initial imperfection starts to increase and grows indefinitely with the external compressive forces (for the elastic plate) of with the time (for the viscoelastic plate) under considerable fixed finite values of external compressive forces. From this criterion we will determine the critical force or the critical time.

The examinations which are not detailed here indicate that the values of critical force or of the critical time can be determined only within the framework of the zeroth and first approximations. Because the accounting of the second and the subsequent approximations do not change the values of the critical parameters. Taking these approximations into account improves only the accuracy of the stress distribution in the plate. Since our aim is to investigate the stability loss (i.e. to determine the values of the critical parameters), we restrict ourselves with the consideration of the zeroth and the first approximations.

According to the foregoing considerations, the stresses in the zeroth approximation have been already determined by the expressions (14). Now we consider the determination of the values of the first approximation for which it is necessary to solve the problem (15)-(19). For this purpose we use the principle of the correspondence by using the Laplace transform

$$\overline{\varphi}(s) = \int_0^\infty \varphi(t)e^{-st}dt \tag{20}$$

with the parameter $s > 0$ to the equations (15)-(19). So, replacing $\sigma_{ij}^{(1)}$, $\varepsilon_{ij}^{(1)}$, $u_i^{(1)}$ and A_{ij} in (15)-(19) by $\overline{\sigma}_{ij}^{(1)}$, $\overline{\varepsilon}_{ij}^{(1)}$, $\overline{u}_i^{(1)}$ and $\overline{A}_{ij}$ respectively, we obtain the corresponding equations and boundary conditions with respect to the Laplace transform for the values of the first approximation. For this purpose we employ FEM and for the FEM modelling of the problem we use the functional

$$\Pi=\frac{1}{2}\iiint\limits_{\Omega-\Omega'}\Bigg[\left(\bar{\sigma}_{11}^{(1)}+\sigma_{11}^{(0)}\frac{\partial\bar{u}_1^{(1)}}{\partial x_1}\right)\frac{\partial\bar{u}_1^{(1)}}{\partial x_1}+\left(\bar{\sigma}_{12}^{(1)}+\sigma_{11}^{(0)}\frac{\partial\bar{u}_2^{(1)}}{\partial x_1}\right)\frac{\partial\bar{u}_1^{(1)}}{\partial x_2}+\bar{\sigma}_{21}^{(1)}\frac{\partial\bar{u}_2^{(1)}}{\partial x_1}+\bar{\sigma}_{22}^{(1)}\frac{\partial\bar{u}_2^{(1)}}{\partial x_2}+$$
$$\left(\bar{\sigma}_{13}^{(1)}+\sigma_{11}^{(0)}\frac{\partial\bar{u}_3^{(1)}}{\partial x_1}\right)\frac{\partial\bar{u}_1^{(1)}}{\partial x_3}+\bar{\sigma}_{31}^{(1)}\frac{\partial\bar{u}_3^{(1)}}{\partial x_1}+\bar{\sigma}_{32}^{(1)}\frac{\partial\bar{u}_3^{(1)}}{\partial x_2}+\bar{\sigma}_{23}^{(1)}\frac{\partial\bar{u}_2^{(1)}}{\partial x_3}+\bar{\sigma}_{33}^{(1)}\frac{\partial\bar{u}_3^{(1)}}{\partial x_3}\Bigg]dx_1dx_2dx_3-$$
$$\int\limits_0^{a_3}\int\limits_{(a_1-a_0)/2}^{(a_1+a_0)/2}\frac{1}{s}\sigma_{11}^{(0)}\frac{\partial f^-}{\partial x_1}\bar{u}_1^{(1)}\Bigg|_{x_2=h_A-0}dx_1dx_3-\int\limits_0^{a_3}\int\limits_{(a_1-a_0)/2}^{(a_1+a_0)/2}\frac{1}{s}\sigma_{11}^{(0)}\frac{\partial f^+}{\partial x_1}\bar{u}_1^{(1)}\Bigg|_{x_2=h_A+0}dx_1dx_3 \qquad (21)$$

where

$$\Omega'=\{(a_1-a_0)/2<x_1<(a_1+a_0)/2,\ x_2=h_A+0,\ 0<x_3<a_3\}\cup$$
$$\{(a_1-a_0)/2<x_1<(a_1+a_0)/2,\ x_2=h_A-0,\ 0<x_3<a_3\} \qquad (22)$$

For the numerical investigations it is used eight-node rectangular brick elements. The number of the finite elements is determined from the convergence requirement of the numerical results. We should note that all computer programs used in the numerical investigations carried out have been composed by the authors in a package FTN77.

After determination of the Laplace transform of the sought values the original of those are determined by employing Schapery method [16].

3. NUMERICAL RESULTS AND DISCUSSION

We assume that the plate material consists of the alternating layers of two isotropic, homogeneous materials and these layers are located in the Ox_1x_3 plane. The reinforcing layers are supposed to be pure elastic with mechanical characteristics $E^{(2)}$ (Young's modulus), $\nu^{(2)}$ (Poisson coefficient). The matrix layers material is supposed to be linearly viscoelastic with operators

$$E^{(1)}=E_0^{(1)}\left[1-\omega_0R_\alpha^*(-\omega_0-\omega_\infty)\right],\ \nu^{(1)}=\nu_0^{(1)}\left[1+\frac{1-2\nu_0^{(1)}}{2\nu_0^{(1)}}\omega_0R_\alpha^*(-\omega_0-\omega_\infty)\right] \qquad (23)$$

where $E_0^{(1)}$, $\nu_0^{(1)}$ are momentary values of the Young's modulus and of the Poisson coefficient, respectively; $\alpha,\omega_0,\omega_\infty$ are the Rheological parameters of the matrix material, R_α^* is the fractional-exponential operator of Rabotnov [17] and this operator is determined as

$$R_\alpha^*\varphi(t)=\int\limits_0^t R_\alpha(\beta,t-\tau)\varphi(\tau)d\tau \qquad (24)$$

where

$$R_\alpha(\beta,t)=t^\alpha\sum_{n=0}^{\infty}\frac{\beta^n t^{n(1+\alpha)}}{\Gamma((1+n)(1+\alpha))},\quad -1<\alpha\le 0. \qquad (25)$$

In equation (25), $\Gamma(x)$ is the Gamma function.

We introduce the dimensionless Rheological parameter $\omega = \omega_\infty / \omega_0$ and the dimensionless time $t' = \omega_0^{1/(1+\alpha)} t$ and assume that $\eta^{(2)} = 0.5$ where $\eta^{(2)}$ is a filler concentration. It is known that in the continuum approach this composite material is taken as viscoelastic, transversaly isotropic material with normalized mechanical properties whose isotropy axis lies on the Ox_2 axis. It is known that these normalized mechanical properties are determined through the well-known operations described, for example, in [18].

As it has been noted above, the sinphase and antiphase initial imperfections are considered. For concrete numerical investigations the sinphase imperfection mode is determined as

$$f^+(x_1) = f^-(x_1) = h_A + a_0 \sin^2\left(\pi\left(x_1 - \frac{a_1 - a_0}{2}\right)\Big/ a_0\right), \tag{26}$$

but the antiphase initial imperfection mode is determined as

$$f^\pm(x_1) = h_A \pm a_0 \sin^2\left(\pi\left(x_1 - \frac{a_1 - a_0}{2}\right)\Big/ a_0\right). \tag{27}$$

Thus, we turn to analysis of the numerical results and first, we consider pure elastic stability loss delamination buckling which takes place at $t' = 0$ and $t' = \infty$. In these cases the reduced critical values of the external compressive force determined as $P_{cr.0} \big/ E_0^{(1)}$ (for $t' = 0$); $P_{cr.\infty} \big/ E_0^{(1)}$ (for $t' = \infty$) are attained for the selected values of the problem parameters.

In the present paper, it is assumed that $h_A = h/2$, $h/a = 0.15$. Thus, we consider the data given in Table 1 which show the values of $P_{cr.0} \big/ E_0^{(1)}$ under $h/a = 0.15$, $a_0/a_1 = 0.5$ for various values of $\gamma = a_3/a_1$ and $E^{(2)} \big/ E_0^{(1)}$. Moreover, the Table 2 shows the ratio of $P_{cr.0}/P_{cr.\infty}$ under $\omega = 1.0$, $E^{(2)} \big/ E_0^{(1)} = 50$ for various γ and a_0/a_1. Note that these data are calculated for the antiphase initial imperfection mode and agree with the corresponding mechanical considerations. Moreover, the trustiness of the numerical results is confirmed by the approaching of those to the corresponding ones obtained in [14] for the plate-strip, i.e. for the case where $t' = \infty$.

Table 1. The values of $P_{cr.0} \big/ E_0^{(1)}$ under $h/a = 0.15$, $a_0/a_1 = 0.5$ for various values of $\gamma = a_3/a_1$ and $E^{(2)} \big/ E_0^{(1)}$.

γ	$E^{(2)} \big/ E_0^{(1)}$			
	1	5	10	50
1	0.0590	0.1497	0.2285	0.4984
1.5	0.0539	0.1365	0.2076	0.4479
2	0.0522	0.1321	0.2007	0.4314
3	0.0511	0.1291	0.1960	0.4200
4	0.0507	0.1280	0.1943	0.4160
5	0.0505	0.1275	0.1935	0.4142
6	0.0504	0.1273	0.1931	0.4132
∞	0.0489	0.1235	0.1904	0.3989

Table 3 shows the values of a ratio $P_{cr.0}/P_{cr.\infty}$ for various initial imperfection modes under $\gamma = 1.0$, $\omega = 1.0$ for the various a_0/a_1, $E^{(2)}/E_0^{(1)}$. According to these results, we can conclude that the values of $P_{cr.0}$ and $P_{cr.\infty}$ attained for the antiphase imperfection mode are grater than those attained for the sinphase initial imperfection mode. As it can be predicted, the corresponding values of $P_{cr.0}$ ($P_{cr.\infty}$) attained for the considered two imperfection modes approach to each other with increasing a_0/a_1 (Table 3).

Table 2. The values of $P_{cr.0}/P_{cr.\infty}$ under $h/a = 0.15$, $a_0/a_1 = 0.5$ for various values of $\gamma = a_3/a_1$ and $E^{(2)}/E_0^{(1)} = 50$

γ	a_0/a_1		
	0.5	0.4	0.3
1	<u>0.4984</u> 0.2935	<u>0.5381</u> 0.3031	<u>0.5874</u> 0.3152
1.5	<u>0.4479</u> 0.2626	<u>0.5013</u> 0.2816	<u>0.5632</u> 0.3020
2	<u>0.4314</u> 0.2524	<u>0.4890</u> 0.2744	<u>0.5550</u> 0.2975
3	<u>0.4200</u> 0.2453	<u>0.4804</u> 0.2694	<u>0.5492</u> 0.2943
4	<u>0.4160</u> 0.2429	<u>0.4774</u> 0.2677	<u>0.5471</u> 0.2932
5	<u>0.4142</u> 0.2418	<u>0.4760</u> 0.2669	<u>0.5462</u> 0.2926
6	<u>0.4132</u> 0.2412	<u>0.4752</u> 0.2664	<u>0.5457</u> 0.2924

Now we consider some numerical results obtained for the critical time, i.e. for the $t'_{cr.}$. It is well known that under investigation of the viscoelastic stability loss problems, the selected values of the external force P must satisfy the inequality $P_{cr.\infty} < P < P_{cr.0}$. For such values of $P/E_0^{(1)}$ the results attained for $t'_{cr.}$ are given in Table 4 for the case where $E^{(2)}/E_0^{(1)} = 1.0$, $a_0/a_1 = 0.5$, $\gamma = 1.0$, $\alpha = -0.5$. It follows from these results that the values of $t'_{cr.}$ increase with increasing ω and with decreasing $P/E_0^{(1)}$. These results agree with those attained in [11-13] in the qualitative sense.

4. CONCLUSION

In the present paper the approach for the studying of the three-dimensional delamination buckling problem for the rectangular plate made from the elastic and viscoelastic composite material is presented. It is supposed that the plate has a band-crack and the edge-surfaces of that have an initial infinitesimal imperfection. The proposed approach is based on the investigation of the development of these imperfections with an external compressive loading acting along the crack (for elastic composite) or with time (for a viscoelastic composites) by the use of the three-dimensional geometrically non-linear field equations of the theory of viscoelasticity. It is assumed that the ends of the plate are simply supported and two modes (sinphase and antiphase modes) of the edge-surface initial infinitesimal imperfections are considered. The concrete numerical results are obtained for the case, where the crack is in the middle plane of the plate. The corresponding boundary-value problems are solved by employing the Laplace transform, FEM and Schapery method. The concrete numerical results are attained for the case where the material of the plate is consists of the alternating layers of the two isotropic viscoelastic materials. According to the numerical results, we can made the following concrete conclusions:

-the values of the critical parameters approach the corresponding ones with the length of the band of the crack;

-the values of the critical parameters attained for the antiphase mode of the initial imperfection are grater than those attained for the sinphase mode of the initial imperfection;

-the values of the critical parameters decrease with increasing crack length

-the values of the critical time increase with the parameter ω.

Table 3. The values of $P_{cr.0}/P_{cr.\infty}$ under $h/a = 0.15, \gamma = 1.0$ for various values $E^{(2)}/E_0^{(1)}$ and a_0/a_1.

$E^{(2)}/E_0^{(1)}$	imperfection mode	a_0/a_1					
		0.1	0.3	0.5	0.6	0.7	0.8
1	antiphase	0.2370 0.1641	0.1032 0.0759	0.0590 0.0445	0.0481 0.0365	0.0409 0.0312	0.0360 0.0276
	sinphase	0.0646 0.0490	0.0572 0.0432	0.0442 0.0334	0.0383 0.0290	0.0336 0.0256	0.0301 0.0231
5	antiphase	0.4696 0.2721	0.2415 0.1667	0.1497 0.1136	0.1253 0.0981	0.1086 0.0870	0.0968 0.0785
	sinphase	0.1739 0.1393	0.1536 0.1229	0.1195 0.0972	0.1044 0.0862	0.0925 0.0774	0.0837 0.0710
10	antiphase	0.5684 0.3053	0.3392 0.2171	0.2285 0.1635	0.1965 0.1462	0.1738 0.1330	0.1566 0.1221
	sinphase	0.2805 0.2148	0.2476 0.1904	0.1952 0.1542	0.1725 0.1391	0.1546 0.1272	0.1414 0.1181

Table 4. The values of $t_{cr.}$ under $h/a = 0.15, \gamma = 1.0$, $E^{(2)}/E_0^{(1)} = 1$, $\alpha = -0.5$ for various values ω

$P/E_0^{(1)}$	ω		
	0.5	1	2
0.042	0.0089	0.0102	0.0139
0.041	0.0221	0.0276	0.0472
0.040	0.0455	0.0632	0.1523

REFERENCES

1. Hwang, S.F., Mao, C.P., The delamination buckling of single-**fibre** system and interply hybrid composites, **Composite Structures**, 46, 279-287, 1999.
2. Short, G.J., Guild, F.J., Pavier, M.J., The effect of delamination geometry on the compressive failure of composite laminates**, Composites Science and Technology**, 61, 2075-2086, 2001.
3. Arman, Y., Zor, M., Aksoy, S., Determination of critical delamination diameter of laminated composite plates under buckling loads**, Composites Science and Technology**, 66, 2945-2953, 2006.
4. Guz, A.N., **Fundamentals of The Three-Dimensional Theory of Stability of Deformable Bodies**, Berlin, Springer-Verlag, 1999.
5. Guz, A.N., **Fracture Mechanics of Composite Materials in Compression**, Naukova Dumka, Kiev, 1990.
6. Guz, A.N., “Nazarenko, V.M., Fracture mechanics of materials in compression along a crack (review). Highly elastic materials, **International Applied Mechanics**, 25, N:9, 3-32, 1989.
7. Guz, A.N., Nazarenko, V.M., Fracture mechanics of materials in compression along a crack (review). Constructional materials, **International Applied Mechanics**, 25, N:10, 3-19, 1989.
8. Akbarov, S.D., On the three dimensional stability loss problems of elements of constructions fabricated from the viscoelastic composite materials, **Mechanics of Composite Materials**, 34, N:6, 537-544, 1998.
9. Akbarov, S.D., Sisman, T., Yahnioglu, N., On the fracture of the unidirectional composites in compression**, International Journal of Engineering Science**, 35, N:12/13, 1115-1136, 1997.
10. Akbarov, S.D., Yahnioglu, N., The method for investigation of the general theory of stability problems of structural elements fabricated from the viscoelastic composite materials, **Composites, Part B: Engineering**, 32, N: 5, 475-482, 2001.
11. Akbarov, S.D., Rzayev, O.G., Delamination of unidirectional viscoelastic composite materials, **Mechanics of Composite Materials**, 38, N: 1, 17-24, 2002.
12. Akbarov, S.D., Rzayev, O.G., On the delamination of a viscoelastic composite circular plate, **International Applied Mechanics**, 39, N: 3, 368-374, 2003.
13. Akbarov, S.D., Rzayev, O.G., On the buckling of the elastic and viscoelastic composite circular thick plate with a penny-shaped crack, **European Journal of Mechanics A/Solids**, 21, N: 2, 269-279, 2002.
14. Rzayev, O.G., Akbarov, S.D., Local buckling of the elastic and viscoelastic coating around the penny-shaped interface crack, **International Journal of Engineering Science**, 40, 1435-1451, 2002.
15. Hoff, N.J., Buckling and stability, **J. Roy. Aeron. Soc.**, 58, 1, 1954.
16. Schapery, R.A., Approximate methods of transform inversion for viscoelastic stress analyses, **Proc. US Nat. Cong. Appl. ASME**, 1075-1085, 1966.
17. Rabotnov, Yu N., **Elements of Hereditary Mechanics of Solid Bodies**, Nauka, Moskow, 1977 (in Russian).
18. Cristensen, R.M., **Mechanics of Composite Materials**, John Willey and Sons, Int. New York, 1979.