ON THE THEORETICAL STRENGTH LIMIT OF THE LAYERED ELASTIC COMPOSITES IN COMPRESSION

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ABSTRACT

The theoretical strength limit in compression (TSLC) of the composite material with alternating two isotropic homogen elastic layers is studied. The investigation is made within the piecewise homogeneous body model with the use of the Three-dimensional Theory of Elasticity. It is assumed that there is an initial imperfection in the local curving form of the reinforcing layers along the direction which is inclined to layers' direction. As the criterion for determination of the TSLC the case for which aforementioned imperfections start to increase indefinitely, is taken. The numerical results on the TSLC are presented.

Keywords: Composite Material, Compression, Elasticity, Initial Imperfection, TSLC.

1. INTRODUCTION

The determination of the theoretical strength limit in compression (TSLC) of the elastic composites has a great significance in the theoretical as well as in the practical sense. Because the TSLC is the standard (in the sense of the upper limit of the compressive forces) for the failure forces attained within the scope of the various mechanisms of the fracture of the fibrous-layered unidirectional composites in compression. It should be noted that one of the major mechanisms of the fracture of the unidirectional composites under uniaxial compression along the reinforcing elements is the stability loss in material structure (structural instability). This fracture mechanism was proposed in [1, 2]. The results obtained in the papers [1, 2] were included in multivolume monographs [3, 4] of encyclopedia character. The experimental verification of the fracture mechanism [1, 2] was made in [5-7]. By now, numerous theoretical investigations have carried out in this field. The review of the investigations which have been made in the three-dimensional statement is given in [8]. The consistent consideration of these investigations has been made in [9]. It follows from the consideration [9] that in many cases under theoretical investigation of the above problems, three-dimensional linearized theory of stability (TDLTS) has been used and these problems studied in the framework of the piecewise-homogeneous body model as well as in the framework of the continual approach. Note that, at first, the application of TDLTS for the investigation of composite fracture mechanics in compression was suggested in the paper [10]. In these cases the equations of TDLTS are obtained from the exact non-linear equations of the deformable solid body mechanics by employing the linearization procedure similarly [11].

Let us call the problems examined in the framework of the piecewise-homogeneous body model the first group fracture problems in compression and call the problems investigated in the framework of the continual approach the second group of fracture problems in compression.

The investigations detailed in [8, 9] and related to the above first group have been made with the use of the bifurcation for TDLTS. In these cases, it is assumed that the reinforcing layers or fibers lose stability in the periodical form and the wave-generation parameter $\chi = \pi h/\ell$ is introduced for layered composite materials or $\chi = \pi R/\ell$ for fibrous composite materials, where h is a thickness of the reinforcing layer, R is a radius of fiber cross-section, ℓ is the half-wavelength of the instability form. Further, using the results of the investigations of the dependencies between χ and the critical values of external compressive force, the value of failure force is determined.

However, in the investigations related to the above second group considered, composite materials first, are modeled as a structurally homogeneous orthotropic material with normalized mechanical characteristics, following this procedure, according to [9, 10, 12] the change of the equations of TDLTS written for the above homogeneous orthotropic infinite body is analyzed under the action at “infinity” uniformly distributed compressive forces. In these cases, if the condition of ellipticity of the equations of TDLTS is not satisfied and these equations lose their ellipticity, then it is assumed that the fracture of the considered material occurs. Moreover, in these cases the value of the external forces corresponding to the above type change of the equations of TDLTS are accepted as the theoretical strength limit in compression (TSLC) of the considered composite material and shows the theoretical limit for which the load carrying capacity exhausts completely.

In the paper [13], a method for investigating the fracture (stability loss) problems of unidirectional composites in compression have been given, which use as a criterion, the value of compressive force for which the initial infinitesimal periodical curvature given to the fiber or reinforcing layer starts to increase and keeps growing indefinitely. In these cases the investigations are carried out in the framework of the piecewise-homogeneous body model with the use of the exact three-dimensional geometrical non-linear equations of the deformable solid body mechanics. However, in [13] the method has been proposed for the investigation of the above first group fracture problems in compression for the composites fabricated from the elastic as well as from the viscoelastic composites. The theoretical approach for the investigations of the second group fracture problems in compression (i.e. for the determination of TSLC) of the composites fabricated from elastic or viscoelastic composites is developed in the paper [14]. At the same time, this approach was developed within the framework of the piecewise-homogeneous body model by the use of the TDLTS taking the following considerations into account.

According to the definitions [9, 10, 12] the values of TSLC determined in the framework of the continual approach, must depend only on the normalized mechanical properties of composite material or only on the micromechanical properties of composite material through which these normalized mechanical properties are determined. Taking this into account in the paper [14] as TSLC, the values of forces for pure elastic composites or the period of time for visco-elastic composites is taken, for which the initial infinitesimal local imperfections given for the reinforcing layers start to increase and grow indefinitely and the values of the forces (for pure elastic composites) or the duration of the period of time does not depend on the initial local imperfection form, nor is it a function of the stability loss form parameters, such as the above-written χ . In [14], by direct verification, it is proved that for the pure elastic composites the values of TSLC determined in the framework of the last definition coincide with the corresponding values of the TSLC determined in the framework of the definition [9, 10, 12].

It should be noted that, in the paper [14], the investigations were made for the layered composite consisting of alternating layers of two materials with local curving (initial imperfection). In this case it was assumed that the local imperfection region of an each reinforcing layer is sin-phase and occupies the same interval along the layers. However, in many cases the observation of the structure of the unidirectional composites shows that these initial imperfection regions have a certain moving with respect to each other. Therefore in the present paper the investigations carried out in [14] is developed for the case where the local curving (initial imperfection) zones of the reinforcing layers are moved with respect to each other. More detail describing of this moving will be considered below.

2. FORMULATION OF THE PROBLEM: SOLUTION METHOD AND THE FRACTURE CRITERION

For the simplicity of the consideration we take a composite material consisting of the alternating layers of two materials with an insignificant local curving (initial local imperfection) in the structure, as shown in Fig.1, under compression along the reinforcing layers at "infinity" by uniformly distributed normal forces of intensity p. The reinforcing layers will be assumed to be located in planes which are parallel to the plane Ox_1x_3 and the thickness of every filler layer will be assumed constant. Moreover, it is assumed that the local imperfection regions of the reinforcing layers are moved with respect to each other in the direction of the Ox_1 axis. This moving length is $a_0 = 2\left(H^{(1)} + H^{(2)}\right)\mathrm{tg}\beta$, where $2H^{(2)}\left(2H^{(1)}\right)$ is a thickness of the reinforcing (matrix) layer, the angle β is shown in Fig.1. Below values related to the matrix will be denoted by upper indices (1); and those related to the filler, by upper indices (2).

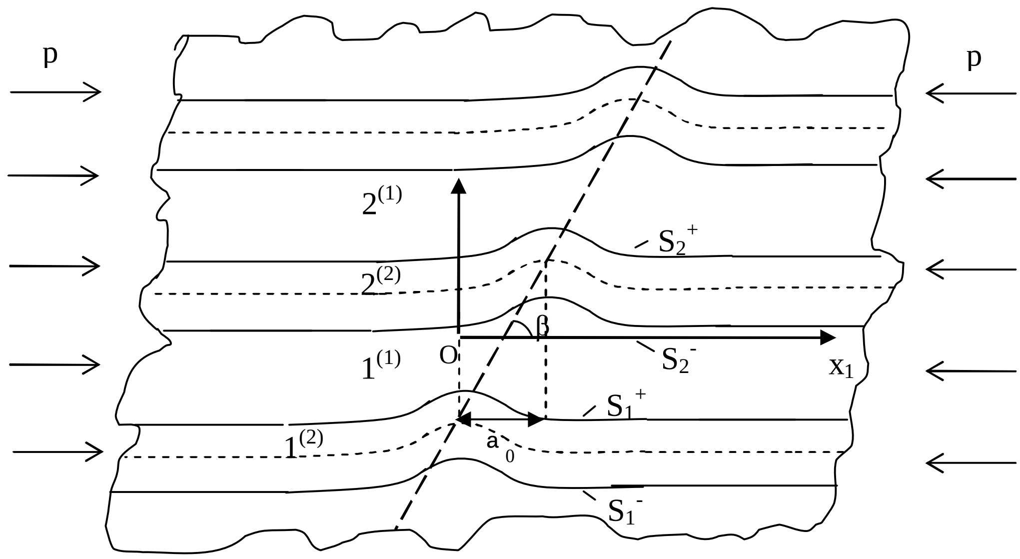

Figure 1. A structure of the considered composite material with inclined local insignificant imperfection.

Taking into account the character of the composite structure shown in Fig.1, among the layers considered we single out three of them, i.e. $1^{(1)}$, $1^{(2)}$, $2^{(2)}$ and discuss them below. We associate the corresponding Lagrangian coordinates $O_m^{(k)}x_{1m}^{(k)}x_{2m}^{(k)}x_{3m}^{(k)}$, where $x_1 = x_{11}^{(1)} = x_{11}^{(2)} = x_{12}^{(2)}$, $x_3 = x_{31}^{(1)} = x_{31}^{(2)} = x_{32}^{(2)}$, although we consider the plane strain state in the Ox_1x_2 plane.

Note that in the natural state these coordinates coincide with Cartesian coordinates and are obtained from $Ox_1x_2x_3$ (Fig.1) by parallel transfer along the Ox_2 axis, with the middle surface of each layer of the filler and matrix.

We investigate the development of the initial insignificant local imperfections (curving) of the filler layers with increasing of the external compressive forces with intensity p (Fig.1).

This investigation will be made within the framework of the piecewise homogeneous body model by the use of the geometrically non-linear exact equilibrium equations of the theory of elasticity. Thus, for each selected layer, we write the equilibrium equations, mechanical and geometrical relations as follows.

$$\frac{\partial}{\partial x_{jm}^{(k)}}\left[\sigma_{jn}^{m(k)}\left(\delta_i^n+\frac{\partial u_{im}^{(k)}}{\partial x_{nm}^{(k)}}\right)\right]=0\ ;\ \ \mathrm{i,\,j,\,n}=1,2\ ;\ \ \mathrm{k}=1,2$$

$$\sigma_{ij}^{m(k)}=\lambda^{(k)}\theta^{m(k)}\delta_i^j+2\mu^{(k)}\varepsilon_{ij}^{m(k)}\quad,\quad \theta^{m(k)}=\varepsilon_{11}^{m(k)}+\varepsilon_{22}^{m(k)}$$

$$2\varepsilon_{ij}^{m(k)}=\frac{\partial u_i^{m(k)}}{\partial x_{j\underline{m}}}+\frac{\partial u_j^{m(k)}}{\partial x_{i\underline{m}}}+\frac{\partial u_n^{m(k)}}{\partial x_{i\underline{m}}^{(k)}}\frac{\partial u_n^{m(k)}}{\partial x_{j\underline{m}}^{(k)}} \tag{1}$$

In (1) and below, repeated indices are summed over their ranges; however, underlined repeated indices are not summed. Moreover, in (1) the conventional notation is used.

We assume that, between the filler and matrix layers there is complete cohesion.

The upper surface of the $2^{(2)}$ $\left(1^{(2)}\right)$ layer we denote by S_2^+ $\left(\mathrm{S}_1^+\right)$, the lower surface by S_2^- $\left(\mathrm{S}_1^-\right)$. The initial insignificant local curvings of the selected filler layers $2^{(2)}$ and $1^{(2)}$ are given through the equation of the middle surface of these layers as

$$x_{22}^{(2)}=\varepsilon\,\mathrm{f}_2(x_1)\ \text{ and }\ x_{21}^{(2)}=\varepsilon\,\mathrm{f}_1(x_1) \tag{2}$$

Here ε is a dimensionless small parameter $(0<\varepsilon<<1)$ the geometric meaning of which is described by the specifically prescribed form of the functions $f_2(x_1)$ and $f_1(x_1)$. According to Fig.1, we can write that

$$f_1(x_1)=f_2(x_1+a_0)\ \text{ or }\ f_2(x_1)=f_1(x_1-a_0) \tag{3}$$

It is also supposed that the functions $f_1(x_1)$ and $f_2(x_1)$ and their first order derivatives are continuous and

$$|f_1(x_1)|\to 0\ ,\ |f_2(x_1)|\to 0\ ,\ |df_2/dx_1|\to 0\ ,\ |df_1/dx_1|\to 0\ \text{ as } |x_1|\to\infty \tag{4}$$

Taking the above stated into account and according to [15], we derive the equations for the surfaces $S_2^{\pm}$ and $S_1^{\pm}$ and the orthonormal components to these surfaces.

For the surfaces $S_2^{\pm}$:

$$x_{i2}^{(2)\pm}=x_{i2}^{(2)\pm}\left(t_1, H^{(2)},\ \varepsilon\ f_2(t_1)\right),$$
$$n_{i2}^{\pm}=n_{i2}^{\pm}\left(t_1, H^{(2)},\ \varepsilon\ f_2(t_1)\right). \tag{5}$$

For the surfaces $S_1^{\pm}$:

$$x_{i1}^{(2)\pm} = x_{i1}^{(2)\pm}\left(t_1, H^{(2)},\ \varepsilon\ f_1(t_1)\right),$$
$$n_{i1}^{\pm} = n_{i1}^{\pm}\left(t_1, H^{(2)},\ \varepsilon\ f_1(t_1)\right), \tag{6}$$

Where t_1 is parameter, $t_1 \in (-\infty, +\infty)$, and $H^{(2)}$ is the half thickness of the $2^{(2)}$ and $1^{(2)}$ the filler layers. The obvious form of the Eqs. (5), (6) is given in [15].

According to (3), we can write

$$x_{i2}^{(2)\pm}\left(t_1 - \alpha_0,\ H^{(2)},\ \varepsilon\ f_2(t_1 - \alpha_0)\right) = x_{i1}^{(2)\pm}\left(t_1, H^{(2)},\ \varepsilon\ f_1(t_1)\right),$$
$$n_{i2}^{\pm}\left(t_1 - \alpha_0,\ H^{(2)},\ \varepsilon\ f_2(t_1 - \alpha_0)\right) = n_{i1}^{\pm}\left(t_1, H^{(2)},\ \varepsilon\ f_1(t_1)\right). \tag{7}$$

Using the equations (5), (6) and (7) we write the contact conditions among the layers $1^{(2)}, 1^{(1)}$ and $2^{(2)}$.

$$\left[\left\{\left(\delta_i^n + \frac{\partial u_i^{1(1)}}{\partial x_{n1}^{(1)}}\right)\sigma_{jn}^{1(1)}\right\}\left(x_{12}^{(2)-}\left(t_1, H^{(2)}, \varepsilon f_2(t_1)\right), x_{22}^{(2)-}\left(t_1, H^{(2)}, \varepsilon f_2(t_1)\right)\right)\right] n_{j2}^{-}\left(t_1, H^{(2)}, \varepsilon f_2(t_1)\right) =$$
$$\left[\left\{\left(\delta_i^n + \frac{\partial u_i^{2(2)}}{\partial x_{n2}^{(2)}}\right)\sigma_{jn}^{2(2)}\right\}\left(x_{12}^{(2)-}\left(t_1, H^{(2)}, \varepsilon f_2(t_1)\right), x_{22}^{(2)-}\left(t_1, H^{(2)}, \varepsilon f_2(t_1)\right)\right)\right] n_{j2}^{-}\left(t_1, H^{(2)}, \varepsilon f_2(t_1)\right),$$
$$u_i^{(1)1}\left(x_{12}^{(2)-}\left(t_1, H^{(2)}, \varepsilon f_2(t_1)\right), x_{22}^{(2)-}\left(t_1, H^{(2)}, \varepsilon f_2(t_1)\right)\right) = u_i^{(2)2}\left(x_{12}^{(2)-}\left(t_1, H^{(2)}, \varepsilon f_2(t_1)\right), x_{22}^{(2)-}\left(t_1, H^{(2)}, \varepsilon f_2(t_1)\right)\right),$$
$$\left[\left\{\left(\delta_i^n + \frac{\partial u_i^{1(1)}}{\partial x_{n1}^{(1)}}\right)\sigma_{jn}^{1(1)}\right\}\left(x_{11}^{(2)+}\left(t_1, H^{(2)}, \varepsilon f_1(t_1)\right), x_{21}^{(2)+}\left(t_1, H^{(2)}, \varepsilon f_1(t_1)\right)\right)\right] n_{j1}^{+}\left(t_1, H^{(2)}, \varepsilon f_1(t_1)\right) =$$
$$\left[\left\{\left(\delta_i^n + \frac{\partial u_i^{1(2)}}{\partial x_{n1}^{(2)}}\right)\sigma_{jn}^{1(2)}\right\}\left(x_{11}^{(2)+}\left(t_1, H^{(2)}, \varepsilon f_1(t_1)\right), x_{21}^{(2)+}\left(t_1, H^{(2)}, \varepsilon f_1(t_1)\right)\right)\right] n_{j1}^{+}\left(t_1, H^{(2)}, \varepsilon f_1(t_1)\right),$$
$$u_i^{1(1)}\left(x_{11}^{(2)+}\left(t_1, H^{(2)}, \varepsilon f_1(t_1)\right), x_{21}^{(2)+}\left(t_1, H^{(2)}, \varepsilon f_1(t_1)\right)\right) = u_i^{1(2)}\left(x_{11}^{(2)+}\left(t_1, H^{(2)}, \varepsilon f_1(t_1)\right), x_{21}^{(2)+}\left(t_1, H^{(2)}, \varepsilon f_1(t_1)\right)\right),$$

$$\left[\left\{\left(\delta_i^n + \frac{\partial u_i^{1(2)}}{\partial x_{n1}^{(2)}}\right)\sigma_{jn}^{1(2)}\right\}\left(x_{11}^{(2)+}\left(t_1, H^{(2)}, \varepsilon f_1(t_1)\right), x_{21}^{(2)+}\left(t_1, H^{(2)}, \varepsilon f_1(t_1)\right)\right)\right] n_{j1}^{+}\left(t_1, H^{(2)}, \varepsilon f_1(t_1)\right) =$$
$$\left[\left\{\left(\delta_i^n + \frac{\partial u_i^{2(2)}}{\partial x_{n2}^{(2)}}\right)\sigma_{jn}^{2(2)}\right\}\left(x_{12}^{(2)}\left(t_1 - \alpha_0, H^{(2)}, \varepsilon f_2(t_1 - \alpha_0)\right), x_{22}^{(2)}\left(t_1 - \alpha_0, H^{(2)}, \varepsilon f_2(t_1 - \alpha_0)\right)\right)\right] \times$$
$$n_{j2}^{+}\left(t_1 - \alpha_0, H^{(2)}, \varepsilon f_2(t_1 - \alpha_0)\right),$$
$$u_i^{1(2)}\left(x_{11}^{(2)+}\left(t_1, H^{(2)}, \varepsilon f_1(t_1)\right),\ x_{21}^{(2)+}\left(t_1, H^{(2)}, \varepsilon f_1(t_1)\right)\right) =$$
$$u_i^{2(2)}\left(x_{12}^{(2)+}\left(t_1 - \alpha_0, H^{(2)}, \varepsilon f_2(t_1 - \alpha_0)\right),\ x_{22}^{(2)+}\left(t_1 - \alpha_0, H^{(2)}, \varepsilon f_2(t_1 - \alpha_0)\right)\right) \tag{8}$$

Thus, with the above-stated the formulation of the problem is exhausted. Now we consider the solution procedure of this problem. Values characterizing the stress-deformation state of the selected layers are sought in series form in the parameter ε

$$\left\{\sigma_{ij}^{m(k)};\ \varepsilon_{ij}^{m(k)};\ u_i^{m(k)}\right\} = \sum_{q=0}^{\infty} \varepsilon^q \left\{\sigma_{ij}^{m(k),q};\ \varepsilon_{ij}^{m(k),q};\ u_i^{m(k),q}\right\} \tag{9}$$

Considering the expressions (5) and (6) we expand the values of each approximation (9) in series in the vicinity of $\left(t_1, \pm H^{(2)}\right)$ and after some operations we obtain from (8) for each approximation the corresponding contact relations which associated with the q-th approximation contain the values of all the previous approximations. Substituting (9) into (1) and comparing equal powers of ε to describe each approximation, we obtain the corresponding closed system equations. Owing to the linearity of the mechanical relations considered, they will be satisfied for every approximation (9) separately. The remaining relations obtained from (1) for every q-th approximation contains the values of all the previous approximations.

Note that the equations and contact conditions for each approximation (9) are given in [13, 15]. Moreover, in [13, 15] it has been shown that the values of the zeroth approximation are determined from nonlinear Eq. (1), the values of subsequent approximations are determined from the linear equations. By direct verification, it is proven that these linear equations are the equations of TDLTS.

Consider the determination of the zeroth and first approximations. It follows from the investigations [13, 15], that under determination of the zeroth approximation the non-linear terms in (1) can be neglected. Therefore, for the zeroth approximation we obtain

$$\sigma_{11}^{(1),0}=p\left(\eta^{(1)}+\eta^{(2)}\frac{1-\left(\nu^{(1)}\right)^2}{1-\left(\nu^{(2)}\right)^2}\frac{E^{(2)}}{E^{(1)}}\right)^{-1};$$

$$\sigma_{12}^{(k),0}=\sigma_{22}^{(k),0}=0\ ;\ u_1^{m(k),0}=\frac{1-\left(\nu^{(k)}\right)^2}{E^{(k)}}\sigma_{11}^{(k),0}x_1$$

$$\sigma_{11}^{(2),0}=\frac{E^{(2)}}{E^{(1)}}\frac{1-\left(\nu^{(1)}\right)^2}{1-\left(\nu^{(2)}\right)^2}\sigma_{11}^{(1),0}\ ,\ C^{m(k)}=const.\ ;$$

$$\eta^{(k)}=\frac{H^{(k)}}{H^{(1)}+H^{(2)}},\ u_2^{m(k),0}=-\frac{\nu^{(k)}\left(1+\nu^{(k)}\right)}{E^{(k)}}x_{2m}^{(k)}+C^{m(k)}$$

$$\sigma_{ij}^{1(k),0}=\sigma_{ij}^{2(k),0}=\sigma_{ij}^{(0)} \tag{10}$$

Now consider the determination of the first order approximation where we obtain the following linearized equations from (1).

The equilibrium equations:

$$\frac{\partial\sigma_{11}^{\underline{m}(\underline{k}),1}}{\partial x_{1\underline{m}}^{(\underline{k})}}+\frac{\partial\sigma_{12}^{\underline{m}(\underline{k}),1}}{\partial x_{2\underline{m}}^{(\underline{k})}}+\sigma_{11}^{(\underline{k}),0}\frac{\partial^2 u_1^{\underline{m}(\underline{k})}}{\partial\left(x_{1\underline{m}}^{(\underline{k})}\right)^2}=0\ ,$$

$$\frac{\partial\sigma_{12}^{\underline{m}(\underline{k}),1}}{\partial x_{1\underline{m}}^{(\underline{k})}}+\frac{\partial\sigma_{22}^{\underline{m}(\underline{k}),1}}{\partial x_{2\underline{m}}^{(\underline{k})}}+\sigma_{11}^{(\underline{K}),0}\frac{\partial^2 u_2^{\underline{m}(\underline{k})}}{\partial\left(x_{1\underline{m}}^{(\underline{k})}\right)^2}=0. \tag{11}$$

The mechanical relations:

$$\sigma_{11}^{\underline{m}(\underline{k}),1}=\frac{E^{(\underline{k})}}{\left(1+\nu^{(\underline{k})}\right)\left(1-2\nu^{(\underline{k})}\right)}\left[\left(1-\nu^{(\underline{k})}\right)\frac{\partial u_1^{\underline{m}(\underline{k}),1}}{\partial x_{1\underline{m}}^{(\underline{k})}}+\nu^{(\underline{k})}\frac{\partial u_2^{\underline{m}(\underline{k}),1}}{\partial x_{2\underline{m}}^{(\underline{k})}}\right];$$

$$\sigma_{22}^{\underline{m}(\underline{k}),1}=\frac{E^{(\underline{k})}}{\left(1+\nu^{(\underline{k})}\right)\left(1-2\nu^{(\underline{k})}\right)}\left[\nu^{(\underline{k})}\frac{\partial u_1^{\underline{m}(\underline{k}),1}}{\partial x_{1\underline{m}}^{(\underline{k})}}+\left(1-\nu^{(\underline{k})}\right)\frac{\partial u_2^{\underline{m}(\underline{k}),1}}{\partial x_{2\underline{m}}^{(\underline{k})}}\right];$$

$$\sigma_{11}^{\underline{m}(\underline{k}),1}=\frac{E^{(\underline{k})}}{\left(1+\nu^{(\underline{k})}\right)}\left[\frac{\partial u_1^{\underline{m}(\underline{k}),1}}{\partial x_{1\underline{m}}^{(\underline{k})}}+\frac{\partial u_2^{\underline{m}(\underline{k}),1}}{\partial x_{2\underline{m}}^{(\underline{k})}}\right]\ , \tag{12}$$

The contact conditions:

$$\sigma_{i2}^{1(1),1}\left(t_1,H^{(1)}\right)-\sigma_{i2}^{2(2),1}\left(t_1,-H^{(2)}\right)=\frac{df_2}{dx_1}\Big|_{x_1=t_1}\left(\sigma_{11}^{(1),0}-\sigma_{11}^{(2),0}\right)\delta_i^1 ,$$
$$u_i^{1(1),1}\left(t_1,H^{(1)}\right)-u_i^{2(2),1}\left(t_1,-H^{(2)}\right)=f_2\big|_{x_1=t_1}\delta_i^2 ,$$
$$\sigma_{i2}^{1(1),1}\left(t_1,-H^{(1)}\right)-\sigma_{i2}^{1(2),1}\left(t_1,H^{(2)}\right)=\frac{df_1}{dx_1}\Big|_{x_1=t_1}\left(\sigma_{11}^{(1),0}-\sigma_{11}^{(2),0}\right)\delta_i^1 ,$$
$$u_i^{1(1),1}\left(t_1,-H^{(1)}\right)-u_i^{1(2),1}\left(t_1,H^{(2)}\right)=f_1\big|_{x_1=t_1}\delta_i^2 ,$$
$$\sigma_{i2}^{1(2),1}\left(t_1-a_0,H^{(2)}\right)=\sigma_{i2}^{2(2),1}\left(t_1,H^{(2)}\right) ,$$
$$u_i^{1(2),1}\left(t_1-a_0,H^{(2)}\right)=u_i^{2(2),1}\left(t_1,H^{(2)}\right) . \tag{13}$$

According to [15], the expressions for the functions $f_2(x_1)$ and $f_1(x_1)$ are selected as follows:

$$x_{22}^{(2)}=\varepsilon f_2(x_1)=A\exp\left\{-\left[\left(\frac{x_1-a_0}{L}\right)^2\right]^{\gamma}\right\}\cos\left(n\frac{x_1-a_0}{L}\right)=\varepsilon L\exp\left\{-\left[\left(\frac{x_1-a_0}{L}\right)^2\right]^{\gamma}\right\}\cos\left(n\frac{x_1-a_0}{L}\right),$$

$$x_{21}^{(2)}=\varepsilon f_1(x_1)=A\exp\left\{-\left[\left(\frac{x_1}{L}\right)^2\right]^{\gamma}\right\}\cos\left(n\frac{x_1}{L}\right)=\varepsilon L\exp\left\{-\left[\left(\frac{x_1}{L}\right)^2\right]^{\gamma}\right\}\cos\left(n\frac{x_1}{L}\right), \tag{14}$$

In (14) the change of the local curving form is characterized by parameters γ, n and L, the geometrical meaning of these parameters is evident. In this case we assume that $A \ll L$ and the small parameter ε is determined by $\varepsilon = A/L$.

Thus, the determination of the values of the first approximation is reduced to the solution of the problem (11)-(14). For solution of this problem we use the exponential Fourier transform

$$f_F(s,x_2)=\int_{-\infty}^{+\infty} f(x_1,x_2)\,e^{-isx_1}dx_1 \tag{15}$$

From the equations (11), (12) we determine the expression of the Fourier transform of the sought values which contain the unknown constants. Denote these constants as $F_1^{1(1)}(s),...F_4^{1(1)}(s)$, $F_1^{1(2)}(s),...F_4^{1(2)}(s)$, $F_1^{2(2)}(s),...F_4^{2(2)}(s)$. For the determination of these constants we obtain the closed inhomogeneous system algebraic equations from the contact conditions (13). According to this statement, the unknown constants can be expressed from the aforementioned system of algebraic equations in the form

$$F_1^{1(1)}(s),...,F_4^{2(2)}(s)=\frac{1}{\det\left\|\alpha_{nm}(s)\right\|}\left(\det\left\|\beta_{nm}^{F_1^{1(1)}(s)}\right\|,...,\det\left\|\beta_{nm}^{F_4^{2(2)}(s)}\right\|\right)\quad n;m=1,2,3,...12 \tag{16}$$

where

$$\alpha_{nm}=\alpha_{nm}\left(sH^{(2)},\eta^{(1)},\eta^{(2)},\mathrm{E}^{(2)}/\mathrm{E}^{(1)},a_0(\beta),\nu^{(1)},\nu^{(2)},p/E^{(1)}\right) \tag{17}$$

are the coefficients of unknowns in the system. Note that the expressions for $\det\left\|\beta_{nm}^{F_1^{2(1)}(s)}\right\|,...,\det\left\|\beta_{nm}^{F_1^{2(2)}(s)}\right\|$ are obtained from $\left\|\alpha_{nm}(s)\right\|$ by replacing the corresponding column of $\left\|\alpha_{nm}(s)\right\|$ with the right side of the algebraic equation system. The explicit forms of the expression α_{nm} are not given here because they are very cumbersome.

Thus, after foregoing preparing procedure, we determine the relation $p = p(sH^{(2)})$ for fixed problem parameters from the equation

$$\det\|\alpha_{nm}\| = 0 \tag{18}$$

It is evident that under determination of the inverse transform

$$f(x_1, x_2) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} f_F(s, x_2)\, e^{isx_1} ds \tag{19}$$

The roots of the equation (18) coincide with the singular points of the integrated function $f_F(s, x_2)$. Consequently, the order the roots of Eq. (18) (denoted by r) is the order of the singularity of the integrated function $f_F(s, x_2)$ in (19). It is known that if $0 \le r < 1$, integral (19) can be calculated by using a well-known algorithm. For $r = 1$, the calculation of the integral is performed in the sense of Cauchy principal value. In the case $r > 1$, the integral (19) does not have any meaning and the force p corresponding to this case, for which the condition

$$\frac{dp}{d(sH^{(2)})} = 0 \tag{20}$$

for the relation $p = p(sH^{(2)})$ determined from the equation (18) is also satisfied, is called the "critical force". According to (20), the critical force corresponds to the local minimum (or maximum) of the function $p = p(sH^{(2)})$ which satisfies Eq. (18). The values of the critical force is taken as the TSLC because it is not depend on the initial local imperfection mode and on the wave generation type parameter such as $\chi = H^{(2)}/L$.

3. NUMERICAL RESULTS

Using the fracture criterion (20) we determine the influence of the angle β (or a_0) (Fig.1) on the values of the TSLC (or p_{cr}) for the various problem parameters $E^{(2)}/E^{(1)}$ and $\eta^{(2)}$. We assume that $\nu^{(1)} = \nu^{(2)} = 0,3$. Under obtaining the numerical results the equation (18) is solved numerically by using PC and by employing "bisection" method.

The Figs.2 and 3 show the dependence between TSLC ($p_{cr}/E^{(1)}$) and angle β for the cases where $E^{(2)}/E^{(1)} = 50$ and 100, respectively, under various values of $\eta^{(2)}$. Note that for the case where $\beta = 0$ the values of $p_{cr}/E^{(1)}$ coincide with the corresponding ones obtained in [8, 9, 14, 16].

It is follows from the graphs given in Figs.2 and 3 that the influence of the β on the values $p_{cr}/E^{(1)}$ decrease with decreasing $\eta^{(2)}$. This result agrees well with the well-known mechanical considerations. Moreover, it is also follows from the graphs that the values of TSLC increase with β and min ($p_{cr}^{(1)}/E^{(1)}$) is attained at $\beta = 0$. This means that the failure of the considered material must be taken place on the plane which is perpendicular on the direction of the compressive force. This concluding is also agreed with the corresponding one attained within the scope of the continuum approach and detailed in [8, 9].

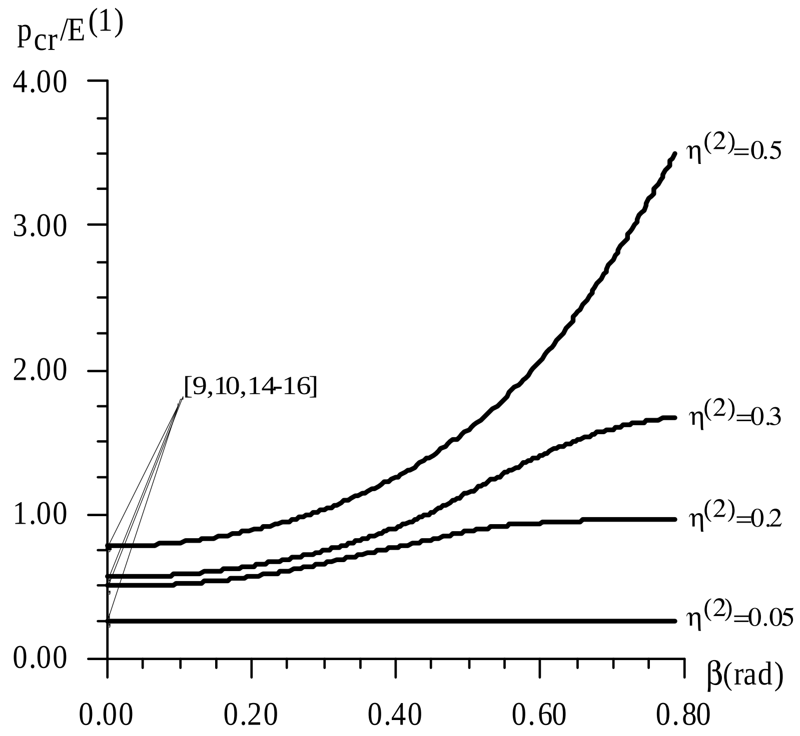

Figure 2. The dependence between TSLC ($p_{cr}^{(1)}/E^{(1)}$) and angle β for $E^{(2)}/E^{(1)}=50$ under various values of $\eta^{(2)}$.

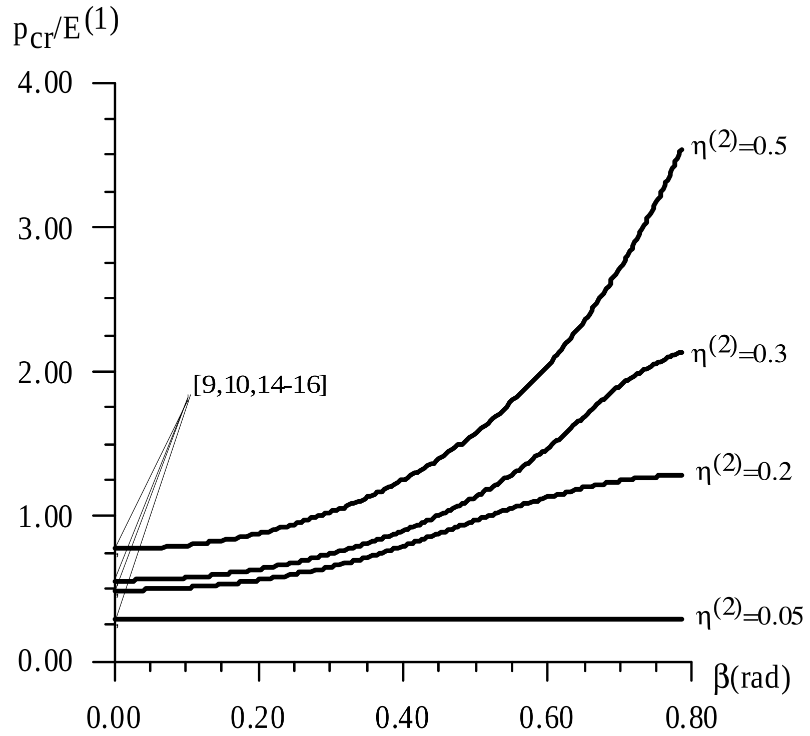

Figure 3. The dependence between TSLC ($p_{cr}^{(1)}/E^{(1)}$) and angle β for $E^{(2)}/E^{(1)}=100$ under various values of $\eta^{(2)}$.

4. CONCLUSION

In the present paper within the framework of the piecewise-homogeneous body model by the use of the exact geometrical non-linear field equations of the theory of elasticity the method is developed for the determination of the TSLC. The corresponding investigations are made on the layered composite consisting of the alternating two layers of the homogeneous isotropic material. It was assumed that the reinforcing layers have the initial local imperfections and these imperfections are moved with each other by the same length. Consequently, the aim of the investigations was to study the influence of this length on the values of the TSLC. In this case this length is expressed by the angle β shown in Fig.1 and as the fracture criterion (i.e. the failure criterion according to which the TSLC is determined) the expression (20) is taken.

The numerical results regarding on the influence of the aforementioned moving of the initial local imperfections on the values of TSLC are presented. According to these results it was established that the values of TSLC increase with length of the foregoing moving.

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