FRACTURE OF AN ORTHOTROPIC PLATE WITH PERIODICAL SYSTEM OF COLLINEAR CRACKS UNDER BIAXIAL LOADING

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ABSTRACT

The fracture of a plate made from an orthotropic material with a periodical system of collinear cracks under biaxial loading is investigated. The cracks are directed along one of the anisotropy axes with external loads being applied in parallelly and perpendicularly to it. The approach to the solution of fracture mechanics problems for orthotropic plate with cracks is suggested on the basis of generalization of Leonov-Panasyuk-Dagdale crack model for the case of orthotropic materials, which satisfy a strength condition of arbitrary form. The influence of the biaxiality of external loading on the critical state of the cracked plate is analyzed within the framework of the critical crack opening criterion.

The analitical expression of the main parameters of crack model (process zone length, stress components in process zone, opening in the crack tip) are received.

The conclusion is made analyzing the critical state of the plate with cracks obtained for the different distance between cracks. The different mechanisms of fracture (confluence of process zone, momentary fracture) are investigated. The influence of external loading biaxiality and orthotropy of material on main parameters of crack model and critical state of plate with periodical cracks system are investigated.

Key words: biaxial loading, orthotropic material, periodical system of collinear cracks.

1. INTRODUCTION

As it is known, some classes of fracture mechanics problems can not be solved within the framework of classic approaches. One of such problems is estimation of the influence which the external load applied along a crack axis exerts on the cracking process. So, in the case of a mode I crack the values of stress intensity factor K_I , J-integral, and crack-tip opening according to the Griffith-Irwin, Cherepanov-Rise, and Leonov-Panasyuk-Dugdale criteria do not depend on the load component applied in parallel to a crack. For this reason the fracture characteristics calculated by the known models do not depend on the loading biaxiality.

To study the problems of such a kind the approach based on the conception of a crack-tip process zone and on the additional necessity to satisfy the strength condition in this zone has been presented in [1].

In present work the problem of fracture of orthotropic plate with a periodical system of collinear cracks under biaxial loading is investigated on base of proposed approach.

2. PROBLEM STATEMENT

Consider a thin orthotropic plate with a crack $|x - x_n| \le l,\ y = 0$, $x_n = \pm 2nD$, oriented along the *Ox*-orthotropy axis and acted upon by the normal loads $\sigma_x^\infty = q$, $\sigma_y^\infty = p > 0$ applied at infinity.

The fracture of the material of the plate is described by the strength criterion in general form

$$F(\sigma_1, \sigma_2, C_i) = 0, \tag{1}$$

where σ_1, σ_2 are the principal stresses, C_i is the material constant.

Based on crack model presented in [1] the boundary-value problem for an orthotropic plate with the slit of length $2L=2(l+d)$ (where d is the length of process zone) under the following boundary conditions can be presented as follows:

$$\sigma_y = \begin{cases} 0, & |x - x_n| < l \\ \sigma_y^0, & l \le |x - x_n| \le L \end{cases}, \ y = 0; \quad \tau_{xy} = 0, -\infty < x < \infty, \ y = 0;$$

$$F(\sigma_x^0, \sigma_y^0, C_i) = 0, \ l \le |x - x_n| \le L, \ y = 0;$$

$$\sigma_x = q, \sigma_y = p, \tau_{xy} = 0, |x + iy| \to \infty; \tag{2}$$

$$v = 0, |x - x_n| \ge L, \ y = 0;$$

3. THE METHOD OF SOLUTION

The boundary-value problem defined by Eq.(2), is solved by its reducing to the Riemann-Gilbert conjugation problem for complex potentials $\omega_1(z_1)$, $\omega_2(z_2)$, where $z_1 = x + S_1 y$; $z_2 = x + S_2 y$; $S_1, S_2, \overline{S_1}, \overline{S_2}$ are the roots of the characteristic equation [2]. For majority of orthotropic materials whose symmetry of elastic properties coincides with coordinate axes, the following holds: $S_1 = i\beta_1$, $S_2 = i\beta_2$, β_1, $\beta_2 \in R$.

Expressions of the stress and displacement components in terms of complex potentials are:

$$\sigma_x = 2\,\mathrm{Re}\left[S_1^2\omega_1(z_1) + S_2^2\omega_2(z_2)\right], \sigma_y = 2\,\mathrm{Re}\left[\omega_1(z_1) + \omega_2(z_2)\right], \tau_{xy} = -2\,\mathrm{Re}\left[S_1\omega_1(z_1) + S_2\omega_2(z_2)\right];$$

$$u = 2\,\mathrm{Re}\left[p_1\Phi_1(z_1) + p_2\Phi_2(z_2)\right], \ v = 2\,\mathrm{Re}\left[q_1\Phi_1(z_1) + q_2\Phi_2(z_2)\right]; \tag{3}$$

where $\Phi_i'(z_i) = \omega_i(z_i)$, $p_i = a_{11}S_i^2 + a_{12}$, $q_i = \dfrac{a_{12}S_i^2 + a_{12}}{S_i}$, $i=1,2$.

By used the transforming [3]

$$w = \sin(\pi z/2D) \tag{4}$$

the plate $z = x + iy$ can be transformed in the plate $w = \tilde{x} + i\tilde{y}$ with one slit $(-\sin(\pi L/2D); \sin(\pi L/2D))$ and the results obtained for plate with one crack in [1] can be used.

The complex potentials $\omega_1(z_1)$, $\omega_2(z_2)$ for problem (2) is determined as

$$\omega_1(w_1)=\frac{1}{\beta_2-\beta_1}\left\{i\frac{\beta_2\sigma_y^0 w_1}{\pi\sqrt{\tilde{L}^2-w_1^2}}\arccos\frac{\tilde{l}}{\tilde{L}}-i\frac{\beta_2 p w_1}{2\sqrt{\tilde{L}^2-w_1^2}}+\right.$$

$$\left.+\frac{\beta_2\sigma_y^0}{2\pi i}\ln\frac{\tilde{L}^2-w_1\tilde{l}+\sqrt{\left(\tilde{L}^2-\tilde{l}^2\right)\left(\tilde{L}^2-w_1^2\right)}}{\tilde{L}^2+w_1\tilde{l}+\sqrt{\left(\tilde{L}^2-\tilde{l}^2\right)\left(\tilde{L}^2-w_1^2\right)}}+\frac{\beta_2\sigma_y^0}{2\pi i}\ln\frac{\tilde{l}+w_1}{\tilde{l}-w_1}-|M_2|\right\},$$

$$\omega_2(w_2)=\frac{1}{\beta_1-\beta_2}\left\{i\frac{\beta_1\sigma_y^0 w_2}{\pi\sqrt{\tilde{L}^2-w_2^2}}\arccos\frac{\tilde{l}}{\tilde{L}}-i\frac{\beta_1 p w_2}{2\sqrt{\tilde{L}^2-w_2^2}}+\right. \qquad (5)$$

$$\left.+\frac{\beta_1\sigma_y^0}{2\pi i}\ln\frac{\tilde{L}^2-w_2\tilde{l}+\sqrt{\left(\tilde{L}^2-\tilde{l}^2\right)\left(\tilde{L}^2-w_2^2\right)}}{\tilde{L}^2+w_2\tilde{l}+\sqrt{\left(\tilde{L}^2-\tilde{l}^2\right)\left(\tilde{L}^2-w_2^2\right)}}+\frac{\beta_1\sigma_y^0}{2\pi i}\ln\frac{\tilde{l}+w_2}{\tilde{l}-w_2}-|M_2|\right\}$$

$$w_1=\tilde{x}+iS_1\tilde{y},\ \ w_2=\tilde{x}+iS_2\tilde{y},\ \tilde{l}=\sin\frac{\pi l}{2D},\ \tilde{L}=\sin\frac{\pi L}{2D},\ |M_2|=\frac{\beta_1\beta_2 p-q}{2(\beta_1+\beta_2)}.$$

4. BASIC PARAMETERS OF THE MODEL

4.1. The stress state of a process zone.

Expressing stresses σ_x in terms of the complex potentials $\omega_1(w_1)$, $\omega_2(w_2)$ and applying the stress continuity condition at the crack front, it can be found:

$$\sigma_x^0=\beta\left(\sigma_y^0-p\right)+q\,. \qquad (6)$$

where $\beta=\sqrt{E_1/E_2}$, E_1,E_2 are the elastic moduli along the orhotropy axes.

Besides, stresses σ_x^0, σ_y^0 satisfy the strength criterion Eq.(1). Eq.(6) together with Eq.(1) form a closed system in the unknown stress components σ_x^0, σ_y^0.

It is evident that stresses within the process zone will be dependent on the elastic constants of a material regardless of the form of the strength criterion Eq.(1).

4.2. The length of a process zone.

The equation to determine of process zone length is received based on condition for restriction of stresses over all the plane ad in considaeted case it can be formed

$$\tilde{l}/\tilde{L}=\cos\left(\pi p/2\sigma_y^0\right) \qquad (7)$$

It is evident that the size of a process zone depends not only on the crack length and external load, but also on elastic orthotropy constants, regarding that the stress σ_y^0 equally depends on these constants.

When the process zone length arrive at value $D-l$ the confluene of adjoining process zone is taked place. So the condition

$$d=D-l \qquad \text{or} \qquad D/l=\frac{\pi}{2\arcsin\left(\cos\left(\pi p/2\sigma_u^0\right)\right)} \qquad (8)$$

can be considered as condition of critical state for the plate with periodic system of collinear cracks.

4.3. Opening in a crack tip.

The opening of the crack in $|x - x_n| \le L,\ y = 0$ can be written down as follows:

$$\delta(x) = -\frac{2T_0\sigma_y^0 l}{\pi}\left\{\frac{\arcsin(\tilde{x})}{\arcsin(\tilde{l})}\ln\frac{\left(1+\frac{\tilde{x}}{\tilde{l}}\cos^2\rho + \sqrt{1-\left(\frac{\tilde{x}}{\tilde{l}}\right)^2\cos^2\rho}\,\sin\rho\right)\left(\frac{\tilde{x}}{\tilde{l}}-1\right)}{\left(1-\frac{\tilde{x}}{\tilde{l}}\cos^2\rho + \sqrt{1-\left(\frac{\tilde{x}}{\tilde{l}}\right)^2\cos^2\rho}\,\sin\rho\right)\left(\frac{\tilde{x}}{\tilde{l}}+1\right)} + \right. \tag{9}$$

$$\left. + 2\frac{\sin\rho}{\arcsin(\tilde{l})}\int_{\tilde{x}/\tilde{l}}^{\sec\rho}\frac{\arcsin(t\tilde{l})}{(t^2-1)\sqrt{1-t^2\cos^2\rho}}dt\right\}$$

where

$$T_0 = \frac{1}{\sqrt{E_1E_2}}\sqrt{2\left(\sqrt{\frac{E_1}{E_2}}-\nu_{12}\right)+\frac{E_1}{G_{12}}}\ ,\ \tilde{x} = \sin\frac{\pi x}{2D},\ \rho = \frac{\pi p}{2\sigma_y^0}.$$

4.4. The critical state of the plate.

In accordance with the δ_c - criterion, a crack starts when the crack tip displacement reaches some ultimate value δ_c , i.e., if

$$\delta(p,q,l) = \delta_c. \tag{10}$$

The fracture of plate with one crack under uniaxial tension ($q = 0$) is discribed by

$$\frac{4T\sigma_y^0\left(p_*^{(1)},0\right)l}{\pi}\ln\sec\frac{\pi p_*^{(1)}}{2\sigma_y^0\left(p_*^{(1)},0\right)} = \delta_c \tag{11}$$

where $p_*^{(1)}$ is the ultimate load in uniaxial tension for one crack.

By comparing Eq.(10) and Eq.(11) and taking into account Eq.(9) it can be found:

$$\sigma_y^0(p_*,q_*)\frac{\tilde{l}}{\arcsin(\tilde{l})}\int_1^{\tilde{L}/\tilde{l}}\ln\left(\frac{1+t\cos^2\rho_* + \sqrt{1-t^2\cos^2\rho_*}\,\sin\rho_*}{1-t\cos^2\rho_* + \sqrt{1-t^2\cos^2\rho_*}\,\sin\rho_*}\frac{t-1}{t+1}\right)\frac{dt}{\sqrt{1-t^2(\tilde{l})^2}} = \tag{12}$$

$$= \sigma_y^0\left(p_*^{(1)},0\right)\ln\sec\frac{\pi p_*^{(1)}}{2\sigma_y^0\left(p_*^{(1)},0\right)}$$

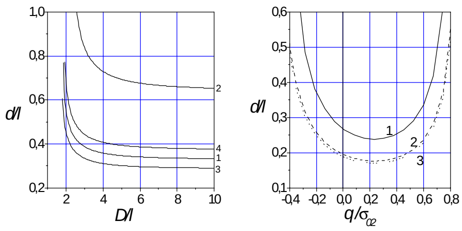
Eq.(12) characterizes the field of ultimate loads (p_*, q_*) depending on the ultimate load value under uniaxial loading. In this case the change in $p_*^{(1)}$ from 0 to σ_{02} (σ_{02} is the ultimate strength in the *y*-direction) corresponds to the change in the crack length from infinity to zero, i.e. it covers all the range of variation in a crack length.

5. RESULTS

Consider as the strength condition Eq.(1) the Mises-Hill criterion which in the case of plane stress is described by

$$\frac{\sigma_x^2}{\sigma_{01}^2}+\frac{\sigma_y^2}{\sigma_{02}^2}-\frac{\sigma_x\sigma_y}{\sigma_{01}\sigma_{02}}=1, \qquad (13)$$

where σ_{01}, σ_{02} are the ultimate strengths in *x*-, *y*-axes direction, respectively. For numerical example the value $\sigma_{01}/\sigma_{02}=0.8$ is used.



a) b)

Figure 1. The process zone length

Figure 1a shows the dimensionnless process zone length d/l depends on dimensionless distance between crack D/l for different values $q/\sigma_{02}=0.0(1)$, -0.3(2), 0.3(3), 0.6(4).

Figure 1b shows the dimensionnless process zone length d/l depends on dimensionless loading q/σ_{02} for different values $D/l=2.0(1)$, 3.0(2), 10.0(3).

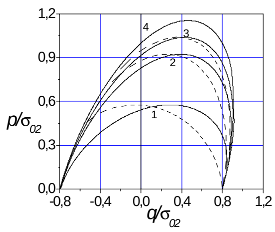


Figure 2. The loading when process zones are connected

Figure 2 shows the strength diagram Eq.(13) (4) and diagram obtained on the basis of Eq.(8) for $D/l=2.0(1)$, 5.0(2), 10.0(3). Here solid lines correspond to $\beta=0{,}3$, dashed ones to $\beta=0{,}9$.

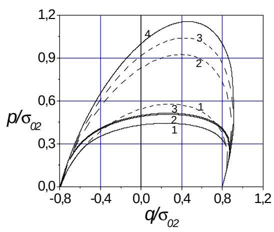


Figure 3. The critical loading.

Figure 3 shows the strength diagram Eq.(11) (4) and the fracture diagrams (solid lines), obtained on the basis of Eq.(12) for $D/l=2.0(1)$, 5.0(2), 10.0(3) under $\beta=0.3$. The dashed lines show external loading when process zones are connected obtained on based of Eq.(8).

6. CONCLUSIONS

The followed conclusion can be made from obtained results. As applied to certain crack length, there are such domains of biaxial loading, where fracture of a cracked plate occurs by fracture of total cross-section on the continuation of a cracsk The load *q* acting along a crack may both increase the ultimate load p_* and decrease it in comparison with uniaxial tension (in perpendicular direction). So, if two materials have the similar strength characteristics but differ in anisotropy level of elastic properties, the crack resistance in the domain of biaxial tension will be higher for the material with a lower ratio E_1 / E_2. In the tension-compression domain the result is opposite to the previous one.

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