INFLUENCE OF THE SCHEME OF FIXING THE SHAFT BORDER-SECTIONS OF THE SHAFT OVER THE FREQUENCY COEFFICIENT OF RESPONSIVENESS TOWARD REGISTERED ONE-SIDE TRANSVERSE SYMMETRICAL CRACKS

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ABSTRACT

Subject of examination in the publication is a steel shaft with registered transverse one-side symmetrical cracks with different location of the crack in progress along the shaft with different degree of penetration of the crack in progress, with different number of the registered cracks and with different accompanying border conditions. The results for the first four own frequencies of the transverse vibrations of the shaft at different degree of penetration of the developing single transverse one-side symmetrical crack with different location along the shaft and at the simultaneous development of two one-side transverse symmetrical cracks with different degree of penetration, are presented. The influence of the degree of penetration of the registered cracks over the relevant own form of the transverse vibrations of the shaft is proved, at different location of the cracks, at different combinations of the simultaneous development of two transverse cracks with different degree of penetration and at different schemes of fixing the border sections of the shaft. Frequency coefficient of responsiveness to the registered cracks is introduced for the examined steel shaft, through which the complex influence of the location of the developing crack, the degree of penetration of the developing crack, the number of the simultaneously registered one-side transverse symmetrical cracks and the imposed restrictions in the field of the border sections over the own frequencies of the transverse vibrations of the shaft and over their relevant own forms, are reported.

Key words: transverse one-side symmetrical cracks; own frequencies of transverse vibrations of elastic element with registered transverse one-side symmetrical cracks; own forms of transverse vibrations of elastic element with registered transverse one-side symmetrical cracks; frequency coefficient of responsiveness toward registered one-side transverse symmetrical cracks.

1. INTRODUCTION

The transverse free vibrations of the elastic beams with registered end transverse symmetrical cracks at different schemes of fixing the border sections are subject to researches implemented by R.D.Adams and others [1978]; A.D.Dimarogonas and others [1970, 1976, 1980, 1983, 1996]; B.S.Haisty and others [1985]; C.A.Papadopoulos and others [1987]; M.H.H. Shen and others [1990 a., 1990 b.]; T.G.Chondros and others [1977, 1979, 1980, 1989];

P.Cawley and others [1979]; N.Anifantis and others [1983]; P.Gudmunson [1984] and Guteva-Veselinova V. [2005, 2006, 2007]. The relevant border tasks are formulated in the publications, which solutions satisfy the system of the partitive differential equations expressing the conditions for dynamic equilibrium, the conditions for compatibility of the deformations and of the tensions within the examined field, and of the borders of the field satisfy the prescribed border conditions reflecting the mode of loading and the mode of fixing. The practical solution of the border tasks in the indicated elaborations is achieved through the method of the finite elements. For this purpose the examined field is dischretizet with different by form and sizes finite elements. Approximate and interpolated functions are proposed, which guarantee similarity of the solution. The functions of the form and the matrix of the form functions are drawing up.

The matrix of the local hardness of the element is introduced and the global matrix of hardness for the discretized field is created, which elements are determined through superposing the matrixes of the elements' hardness as per knots.

Subject of examination in the present elaboration is a steel shaft with registered one-side transverse symmetrical cracks along its length, the position of which is determined through the co-ordinates x_1 and x_2 which are reported from the most left section of the shaft [N.N.Minchev and others-figure 1.- 2005;2007]. The effect from the registered transverse crack in the examined elastic medium is reported as a change of the hardness upon bending and it is assumed that the hardness change obeys a linear law in the field restricted by the effective length $2.l^*$. The geometric parameter l^* is determined by the condition for equivalence of the different approaches for modeling the field containing the registered transverse crack. As a result of modeling the field containing the crack, at the introduced simplifying premise, which guarantees linear law for hardness' change upon bending of the shaft in a plain perpendicular to the plain of the crack development, the conclusion is reached that the parameter l^* does not depend on the depth of penetration of the crack, but depends on the main determining parameter of the cross section of the examined elastic element, in this case on the diameter of the shaft - $l^* \approx 1,5.d$ [N.N.Minchev and others – formula (4) – 2005].

The purpose of the present work is determination of the own frequencies and of their respective own forms of the transverse vibrations for a steel shaft at different location of the developing crack, at different number of the registered cracks along the shaft and at different schemes of fixing the border sections of the shaft. The researches are implemented in a plain perpendicular to the plain of development of the registered cracks, which is characterized with minimal values for shaft hardness upon transverse specific bending. For the practical accomplishment of the tasks, which guarantee the achievement of the formulated objective, created synthesized computer finite-element model of the shaft is used, in the program medium ***NX3.0*** with volumetric finite elements with four knots ***«tetraeder»*** [N.N.Minchev and others – figure 4. – 2005, 2007]. Pertaining to the group of obligatory entrance data, through which the physical-mechanical characteristics of the examined construction material for the shaft ***«steel HGSA»*** are described, are: the module of elasticity of first type $E=2,06.10^{11}[N.m^{-2}]$; the coefficient of Poison $\mu=0,335$ and the density of the material $\rho=7800[kg.m^{-3}]$. Taking into consideration that the modeling of the shaft, of the locality of the registered cracks and of the field of influence of the registered crack is implemented parametrically through the diameter of the shaft $d=50[mm]$, then for the main geometrical sizes of the examined shaft, the following entrance data is summarized: length of the shaft $L=20.d=1000[mm]$; position coordinates of the cracks $x_1=5,6.d=280[mm]$ and $x_2=14,4.d=720\ [mm]$. The degree of penetration of the developing crack is expressed through the shaft diameter $h_j(0)=k_{h,j}(0).d$, as the coefficient of proportionality $\boldsymbol{k_{h,j}(0)}$ in the volume of the conducted researches receives the following numerical values: $\boldsymbol{k_{h,j}(0)=0,10; 0,20; 0,30; 0,40; 0,45}$ ***at j=1,2.*** The main dependencies and data for the geometrical characteristics of the transverse sections and for the introduced parametrical coefficient of responsiveness toward the crack in the field of influence of the registered crack are presented as summary by Guteva-Veselinova and others [figure 1., table 1. and table 2. – 2006].

2. MAIN DEPENDENCIES.

For the examined elastic medium, the equation of movement of the free vibrations without reporting the resistance forces is expressed through the global matrix of hardness ***[C],*** the global mass matrix ***[M]*** and the global vector of the unknown knot shiftings ***{F},*** as the final matrix look of the equation is:

$$[M]\{\ddot{F}\}+[C]\{F\}=0 \tag{1.}$$

which solution is search from the look:

$$\{F\}=\{f_a\}\sin(\omega.t) \tag{2.}$$

where $\{f_a\}$ is global vector of the unknown dynamic amplitudes of the knots. After substitution of the matrix equation (2.) in matrix equation (1.), a new matrix recording of the equation of movement at the free vibrations without reporting the resistance forces, is derived in the look:

$$([C]-\omega^2[M])\{f_a\}=0 \tag{3.}$$

It was ascertained in the linear algebra that one homogenous system algebraic equations has non-zero solution when the determinant from the matrix in front of the unknown is equal to zero, i.e. the non-zero solution for the unknown dynamic amplitudes of the knots is determined by the condition:

$$\det([C]-\omega^2[M])=0 \tag{4.}$$

The roots of the characteristic equation (4.) are the own frequencies of the free vibrations without reporting the resistance forces for the examined elastic medium. The solution of equation (3.) at $\omega=\omega_i$, where $i=1\div n$, determines the deformed condition of the examined elastic medium which corresponds to the relevant own frequency ω_i, i.e. determines the own form of the elastic medium of the free vibrations at the respective own frequency ω_i. Therefore, the determination of the own frequencies for the examined elastic medium at the free vibrations without reporting the resistance forces and of the own forms of vibrations at the respective own frequencies, it is required the global matrix of hardness ***[C]*** and the global matrix ***[M]*** for the examined elastic medium to be determined. This fact necessitates upon the creation of the synthesized computer model for the examined elastic shaft in the program medium ***NX3.0*** with volumetric finite elements with four knots «***tetraeder***», the following main procedures to be performed:

- ***choice of the approximate functions in the field of the volumetric finite element «tetraeder» with four knots «i, j, m, p».*** In order to achieve faster similarity of the method of the finite elements, it is recommended full polynominals to be used. The creation of full polynominal from a given exponent is realized conveniently with the help of the pyramid of Pascal. Then the components of the complete shiftings in the field of the volumetric finite element are determined synonymously with the help of three linear multinominals from the coordinates of the examined point, i.e. the following approximate functions are introduced:

$$\begin{cases} u^e = \alpha_1 + \alpha_2.x + \alpha_3.y + \alpha_4.z \\ v^e = \beta_1 + \beta_2.x + \beta_3.y + \beta_4.z \\ w^e = \gamma_1 + \gamma_2.x + \gamma_3.y + \gamma_4.z \end{cases} \quad (5.)$$

and the vector complete shifting is:

$$\{\tilde{f}^e\} = [u^e(x,y,z) \quad v^e(x,y,z) \quad w^e(x,y,z)]^T. \quad (6.)$$

If we substitute in the equations from the system (5.) the co-ordinates of a point in the field of the element consecutively with the co-ordinates of the knots ***«i, j, m, p»*** of the finite element, a system is drawing up of twelve algebraic equations with twelve unknown coefficients $\{\alpha\} = [\alpha_1 \alpha_2 \alpha_3 \alpha_4]^T$, $\{\beta\} = [\beta_1 \beta_2 \beta_3 \beta_4]^T$ and $\{\gamma\} = [\gamma_1 \gamma_2 \gamma_3 \gamma_4]^T$. After the determination of the coefficients and their back substitution in the equations from system (5.), the following interpolated functions are summarized:

$$\begin{cases} u^e = \varphi_i(x,y,z).u_i + \varphi_j(x,y,z).u_j + \varphi_m(x,y,z).u_m + \varphi_p(x,y,z).u_p \\ v^e = \varphi_i(x,y,z).v_i + \varphi_j(x,y,z).v_j + \varphi_m(x,y,z).v_m + \varphi_p(x,y,z).v_p \\ w^e = \varphi_i(x,y,z).w_i + \varphi_j(x,y,z).w_j + \varphi_m(x,y,z).w_m + \varphi_p(x,y,z).w_p \end{cases} \quad (7.)$$

or in matrix form:

$$\{\tilde{f}^e\} = [\phi^e]\{f^e\} \quad (8.)$$

where:

$$\{f^e\} = [f_i \ f_j \ f_m \ f_p]^T \quad (9.)$$

is the vector complete shiftings of the knots of the finite element, as each vector of them is expressed through the relevant components:

$$\begin{cases} \{f_i\} = [u_i \ v_i \ w_i]^T; \quad \{f_j\} = [u_j \ v_j \ w_j]^T; \\ \{f_m\} = [u_m \ v_m \ w_m]^T; \{f_p\} = [u_p \ v_p \ w_p]^T; \end{cases} \quad (10.)$$

$$\left[\phi^e\right]=\begin{bmatrix}\phi_i & 0 & 0 & \phi_j & 0 & 0 & \phi_m & 0 & 0 & \phi_p & 0 & 0\\ 0 & \phi_i & 0 & 0 & \phi_j & 0 & 0 & \phi_m & 0 & 0 & \phi_p & 0\\ 0 & 0 & \phi_i & 0 & 0 & \phi_j & 0 & 0 & \phi_m & 0 & 0 & \phi_p\end{bmatrix} \tag{11.}$$

is matrix of the functions of the form; as the form functions have the look and satisfy the criteria for continuity of the shiftings, the criterion to provide shifting of the finite element as sold body and the criterion to provide possibility for performance of constant deformation within the limits of the element:

$$\begin{cases}\phi_i(x,y,z)=\dfrac{1}{6.V_T}(a_i+b_i.x+c_i.y+d_i.z)\\ \phi_j(x,y,z)=\dfrac{1}{6.V_T}(a_j+b_j.x+c_j.y+d_j.z)\\ \phi_m(x,y,z)=\dfrac{1}{6.V_T}(a_m+b_m.x+c_m.y+d_m.z)\\ \phi_p(x,y,z)=\dfrac{1}{6.V_T}(a_p+b_p.x+c_p.y+d_p.z)\quad .\end{cases} \tag{12.}$$

The volume of the finite element ***«tetraeder»*** $\boldsymbol{V_T}$ and the coefficients from the equations for the functions of the form are defined as determinant of the following matrixes:

$$6.V_T=\det\begin{bmatrix}1 & x_i & y_i & z_i\\ 1 & x_j & y_j & z_j\\ 1 & x_m & y_m & z_m\\ 1 & x_p & y_p & z_p\end{bmatrix} \tag{13.}$$

$$\begin{cases}a_i=\det\begin{bmatrix}x_j & y_j & z_j\\ x_m & y_m & z_m\\ x_p & y_p & z_p\end{bmatrix};\ b_i=-\det\begin{bmatrix}1 & y_j & z_j\\ 1 & y_m & z_m\\ 1 & y_p & z_p\end{bmatrix};\\ c_i=-\det\begin{bmatrix}x_j & 1 & z_j\\ x_m & 1 & z_m\\ x_p & 1 & z_p\end{bmatrix};\quad d_i=-\det\begin{bmatrix}x_j & y_j & 1\\ x_m & y_m & 1\\ x_p & y_p & 1\end{bmatrix}.\end{cases} \tag{14.}$$

The remaining coefficients of the equations in system (12.) are determined in analogous mode after cyclic change of the indexes. It has to be noted that the adopted approximate functions (5.) secure continuity of the shiftings between the neighboring finite elements, since the shiftings alter linearly along each edge of the volumetric finite element.

- ***drawing up of the matrix of the deformations.*** Upon volumetric strained condition, full tensor of the strained and deformed condition are on hand. According to the equations of Koshi for volumetric deformed condition in the field of the finite element at small shiftings, for the deformations in an unspecified point of the element in matrix form, it should be written down:

$$\{\varepsilon^e\} = [\partial]^T \{f^e\} = [B^e]\{f^e\} \qquad (15.)$$

where:

$$\begin{cases} [B^e] = [B_i \; B_j \; B_m \; B_p]; \\ [B_i]^T = \frac{1}{6.V_T}\begin{bmatrix} b_i & 0 & 0 & c_i & 0 & d_i \\ 0 & c_i & 0 & b_i & d_i & 0 \\ 0 & 0 & d_i & 0 & c_i & b_i \end{bmatrix}; \quad [B_j]^T = \frac{1}{6.V_T}\begin{bmatrix} b_j & 0 & 0 & c_j & 0 & d_j \\ 0 & c_j & 0 & b_j & d_j & 0 \\ 0 & 0 & d_j & 0 & c_j & b_j \end{bmatrix}; \\ [B_m]^T = \frac{1}{6.V_T}\begin{bmatrix} b_m & 0 & 0 & c_m & 0 & d_m \\ 0 & c_m & 0 & b_m & d_m & 0 \\ 0 & 0 & d_m & 0 & c_m & b_m \end{bmatrix}; \quad [B_p]^T = \frac{1}{6.V_T}\begin{bmatrix} b_p & 0 & 0 & c_p & 0 & d_p \\ 0 & c_p & 0 & b_p & d_p & 0 \\ 0 & 0 & d_p & 0 & c_p & b_p \end{bmatrix} \end{cases} \qquad (16.)$$

is matrix of the deformations. As a result of the adopted linear approximation of the shiftings, the elements of the deformations' matrix are constant, and this indicates that the deformations in the field of each finite element are constant.

- ***drawing up of the matrix of the physical constants.*** Upon full tensor of the strained condition, the tensions are connected with the deformations through the summarized law of Huck. If the equations reflecting this relation are solved with respect to the elements of the tensor-tension in matrix recording, the following is derived:

$$\{\sigma^e\} = [D^e]\{\varepsilon^e\} = [D^e][B^e]\{f^e\} \qquad (17.)$$

where:

$$\begin{cases} [D^e] = \frac{E.(1-\mu)}{(1+\mu).(1-2\mu)}\begin{bmatrix} d_{11} & d_{12} & d_{13} & 0 & 0 & 0 \\ & d_{22} & d_{23} & 0 & 0 & 0 \\ & & d_{33} & 0 & 0 & 0 \\ & & & d_{44} & 0 & 0 \\ \text{symmetric} & & & & d_{55} & 0 \\ \text{matrix} & & & & & d_{66} \end{bmatrix}; \quad \begin{aligned} & d_{11} = d_{22} = d_{33} = 1; \\ & d_{12} = d_{13} = d_{23} = \frac{\mu}{1-\mu}; \\ & d_{44} = d_{55} = d_{66} = \frac{1-2\mu}{2.(1-\mu)} \end{aligned} \end{cases} \qquad (18.)$$

is matrix of the elasticity, which characterizes the properties of the isotropic and homogenous material medium, since it reports the module of the elasticity of first type and the coefficient of Poison.

- ***drawing up of the matrix of the element hardness.*** According to Zenkevich O. [1982], the matrix of the element hardness is determined through the matrix of the element deformations $[B^e]$, through transposed matrix of the element deformations $[B^e]^T$ and through the matrix of the elasticity $[D^e]$ with the expression presented in matrix form:

$$[C^e] = \int_{(V_T)} [B^e]^T [D^e][B^e]\, d(vol). \qquad (19.)$$

Since the determining matrixes do not depend on the co-ordinates of an unspecified point of the element, the matrix of the element hardness is summarized:

$$[C^e] = [B^e]^T [D^e][B^e] V_T. \qquad (20.)$$

In developed form, the matrix of the element hardness «*i, j, m, p*» is presented in the form:

$$[C^e] = V_T . \begin{bmatrix} B_i^T D B_i & B_i^T D B_j & B_i^T D B_m & B_i^T D B_p \\ B_j^T D B_i & B_j^T D B_j & B_j^T D B_m & B_j^T D B_p \\ B_m^T D B_i & B_m^T D B_j & B_m^T D B_m & B_m^T D B_p \\ B_p^T D B_i & B_p^T D B_j & B_p^T D B_m & B_p^T D B_p \end{bmatrix} \qquad (21.)$$

or

$$[C^e] = \begin{bmatrix} c_{ii} & c_{ij} & c_{im} & c_{ip} \\ c_{ji} & c_{jj} & c_{jm} & c_{jp} \\ c_{mi} & c_{mj} & c_{mm} & c_{mp} \\ c_{pi} & c_{pj} & c_{pm} & c_{pp} \end{bmatrix} \qquad (22.)$$

where: the sub-matrixes $[C^e_{rs}]$ are presented in the following summarized matrix form:

$$[C^e_{rs}] = [B^e_r]^T [D^e][B^e_s] V_T \quad (r, s = 1,2,3,4). \qquad (23.)$$

The global matrix of hardness for the examined field, which is discretized through $\tilde{N}$ number of finite elements, is determined through superposing the matrixes of the elements' hardness:

$$[C] = \sum_{e=1}^{\tilde{N}} [C^e] \qquad (24.)$$

- ***drawing up of mass matrix of the element.*** This stage of the creation of the synthesized finite element model of the shaft is realized in one of the following two approaches. In the first approach, the mass of the element is allotted on the knots, which border it, by adhering to the only rule the sum of the allotted masses to be equal to the mass of the element. As most frequently upon the practical realization of this approach, the mass of the element is allotted over its knots equally. In the second approach, the mass of the element is allotted over its knots as the continuity and the deformativity of the element are reported. According to equation (8.) the deformativity of the element is described with the introduced functions of the form $[\phi_i\ \phi_j\ \phi_m\ \phi_p]$. This is why Zenkevich O. [1982] offers the allotment of the mass of the element on its knots to be realized the way it is done during the creation of the matrix of the element hardness, as the mass sub-matrix is defined through the expression:

$$[m_{rs}^{e}] = \int_{(V_T)} \phi_r^{T} \rho \phi_s dV \quad (r, s = i, j, m, p) \qquad (25.)$$

The global mass matrix is determined through addition of the harmonized mass matrixes for the elements $\tilde{N}$ through which the examined elastic field is discretized:

$$[M] = \sum_{e=1}^{\tilde{N}} [M^{e}] \qquad (26.)$$

3. RESULTS AND CONCLUSIONS.

In furriers 1., 2. and 3., the results for the first four own frequencies of the transverse vibrations for the examined steel shaft are presented, at registered single transverse one-side symmetrical cracks with different locality along the length of the shaft and with different degree of penetration of the cracks (schemes 1., 1.*, 1.**, and schemes 2., 2.*, 2.**); and at registered two transverse one-side symmetrical cracks with different locality of the developing crack along the shaft and with differenet degrees of penetration of the cracks (schemes 3., 3.*, 3.** and schemes 4., 4.*, 4.**). In figures 4. and 5., the comparative assessment for the alterations of each one of the first four own frequencies of the transverse vibrations for the examined steel shaft is presented at the examined four schemes depending on the locality of the developing crack, the degree of penetration of the developing crack, the number of the registered cracks along the shaft by the examined three way of fixing the border sections of the shaft. From the indicated graphical dependencies which describe the quantity and quality alterations of the first four own frequencies of the transverse vibrations for the examined steel shaft at different locality of the developing crack along the shaft, at different degree of penetration of the developing crack and at different number of the registered cracks along the shaft, it is ascertained that the examined problem is multifactorial and the separate factors have impact in different degree at the different combinations, i.e. the mutual connection between the separate factors is with different power of interaction. This fact necessitated a coefficient to be introduced via which the complex influences of the examined factors to be reported. The introduced coefficient is determined as relation between the own frequency of the transverse vibrations of the examined steel shaft with registered crack with exactly established locality along the shaft, precisely determined depth of penetration and precisely determined border conditions $f_{i,h}$ and the respective own frequency of the transverse vibrations of the examined steel shaft without registered crack $f_{i,0}$, i.e.:

$$\chi_i = \frac{f_{i,h}}{f_{i,0}}; \qquad \chi_i = \frac{f_{i,h}^{*}}{f_{i,0}^{*}}; \qquad \chi_i = \frac{f_{i,h}^{**}}{f_{i,0}^{**}}. \qquad (27.)$$

The designation ***«frequency coefficient of responsiveness toward the registered crack»*** is adopted and the data for it within the framework of the realized research is geometrically presented in figures 6., 7. and 8., and numerical values-in table 1. On the basis of the analysis of the received results the following conclusions are summarized:

♦ upon increase of the degree of penetration of the developing transverse one-side symmetrical crack, irrespective of the accompanying border conditions, the own frequencies of the transverse vibrations of the shaft decrease – figure 1. and figure 5. This is so, since every reg-

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istered crack leads to change to the local hardness of the examined shaft, i.e. to change to the knot shiftings of the finite elements, through which the examined field is discretized, or to change of the field of the shiftings, which determines the new deformed condition of the examined elastic element;

♦ the quantity changes of the own frequencies of the transverse vibrations for the examined shaft depending on the degree of penetration of the developing crack are more strongly expressed at the higher frequencies at a concrete scheme of fixing the border sections of the shaft – figure 1. ÷ figure 5. Therefore, if the change of the own frequencies of the transverse vibrations for the examined steel shaft is used as diagnostic parameter in the methods for non-demolishing control with the purpose the existence of transverse cracks in the shaft to be ascertained, their locality along the shaft to be localized and their degree of penetration to be specified, it is desirable to use the information vibro-diagrams ascertained at the higher frequencies;

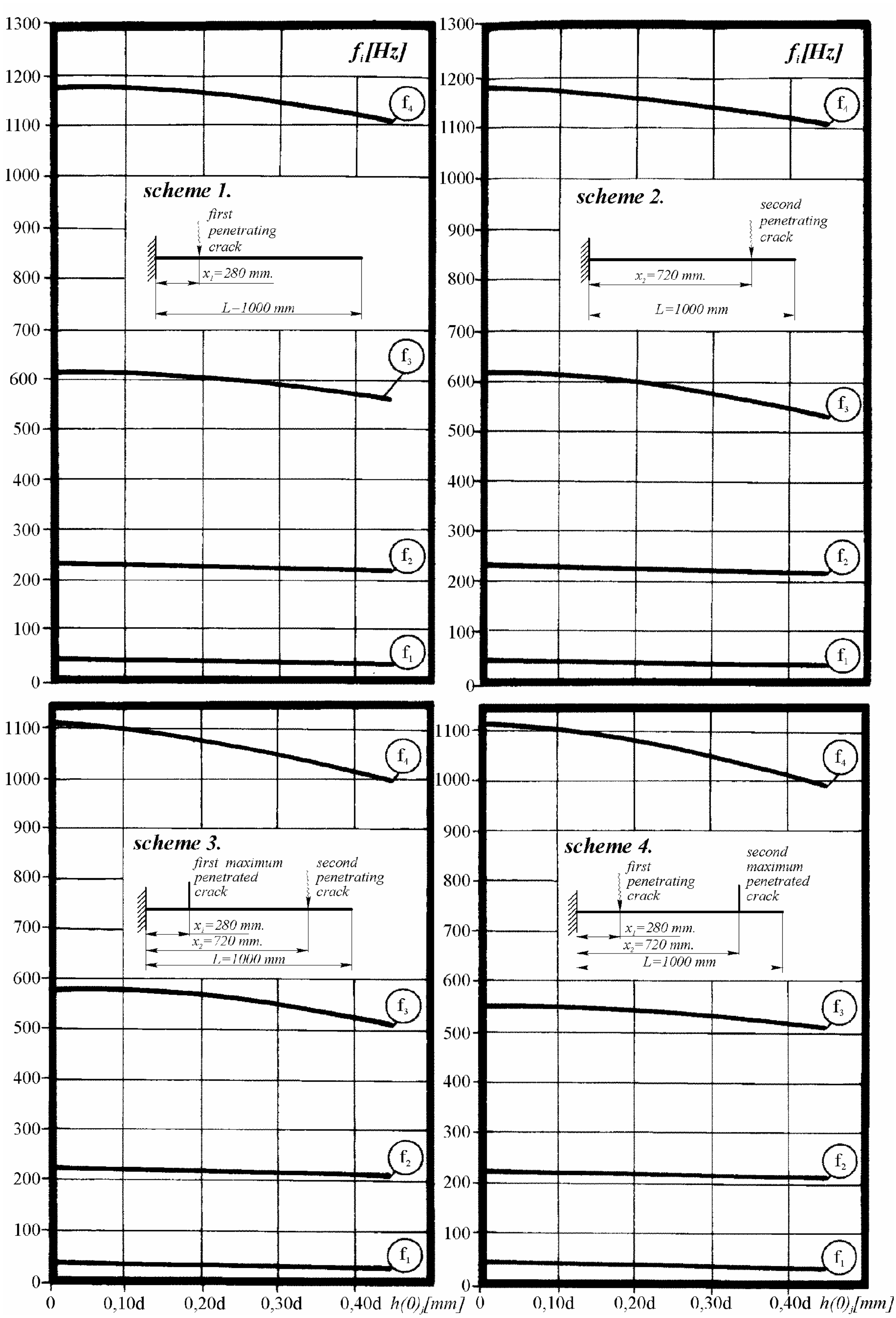

Fig. 1. Dependency of the first four own frequencies f_i of the transverse vibrations for a steel elastic shaft with fixed left border section on the locality x_j and the depth of penetration $h(0)_j$ of the developing transverse one-side symmetrical crack.

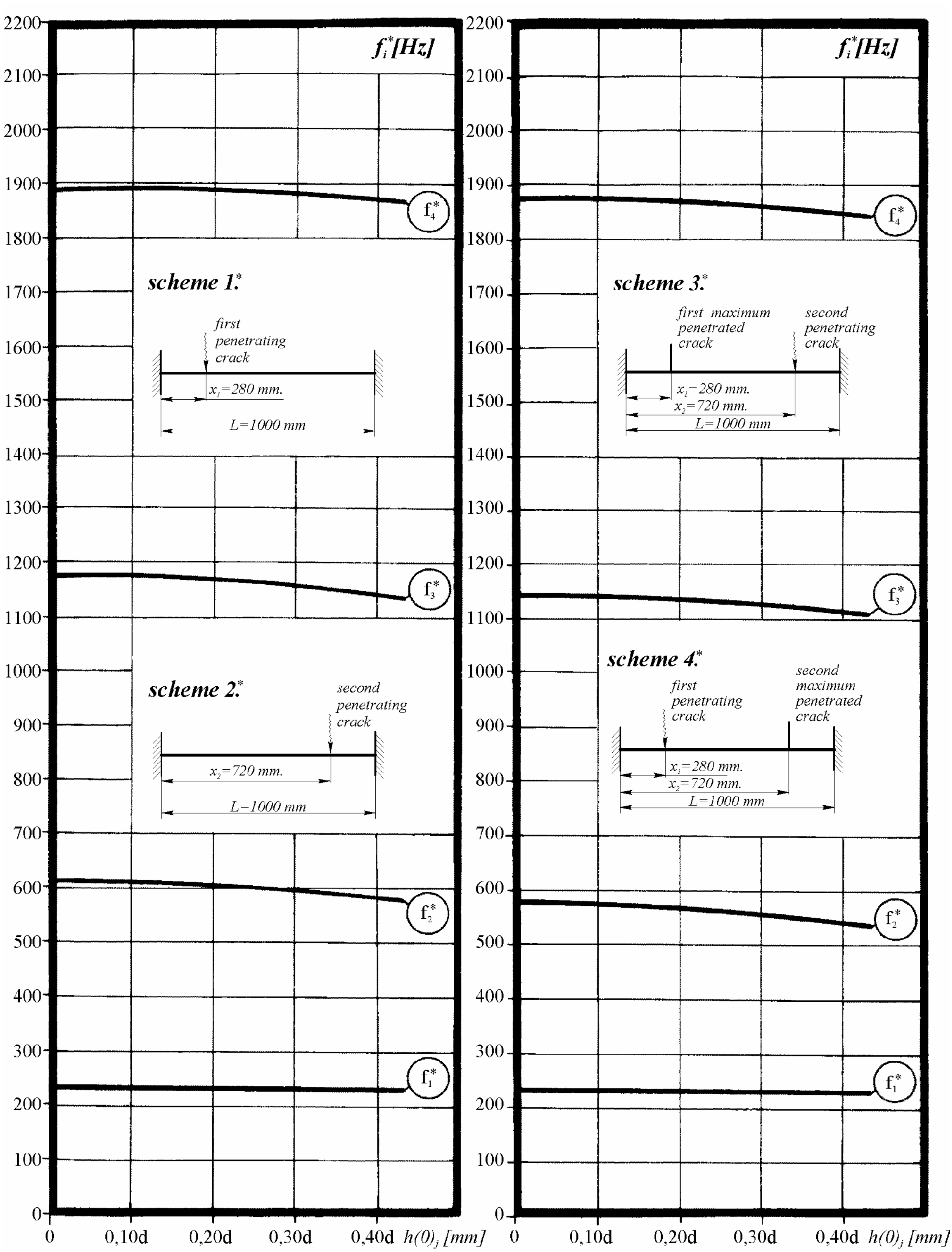

Fig.2. Dependency of the first four own frequencies f_i* of the transverse vibrations for a steel elastic shaft with fixed border sections on the locality x_j and the depth of penetration $h(0)_j$ of the developing transverse one-side symmetrical crack.

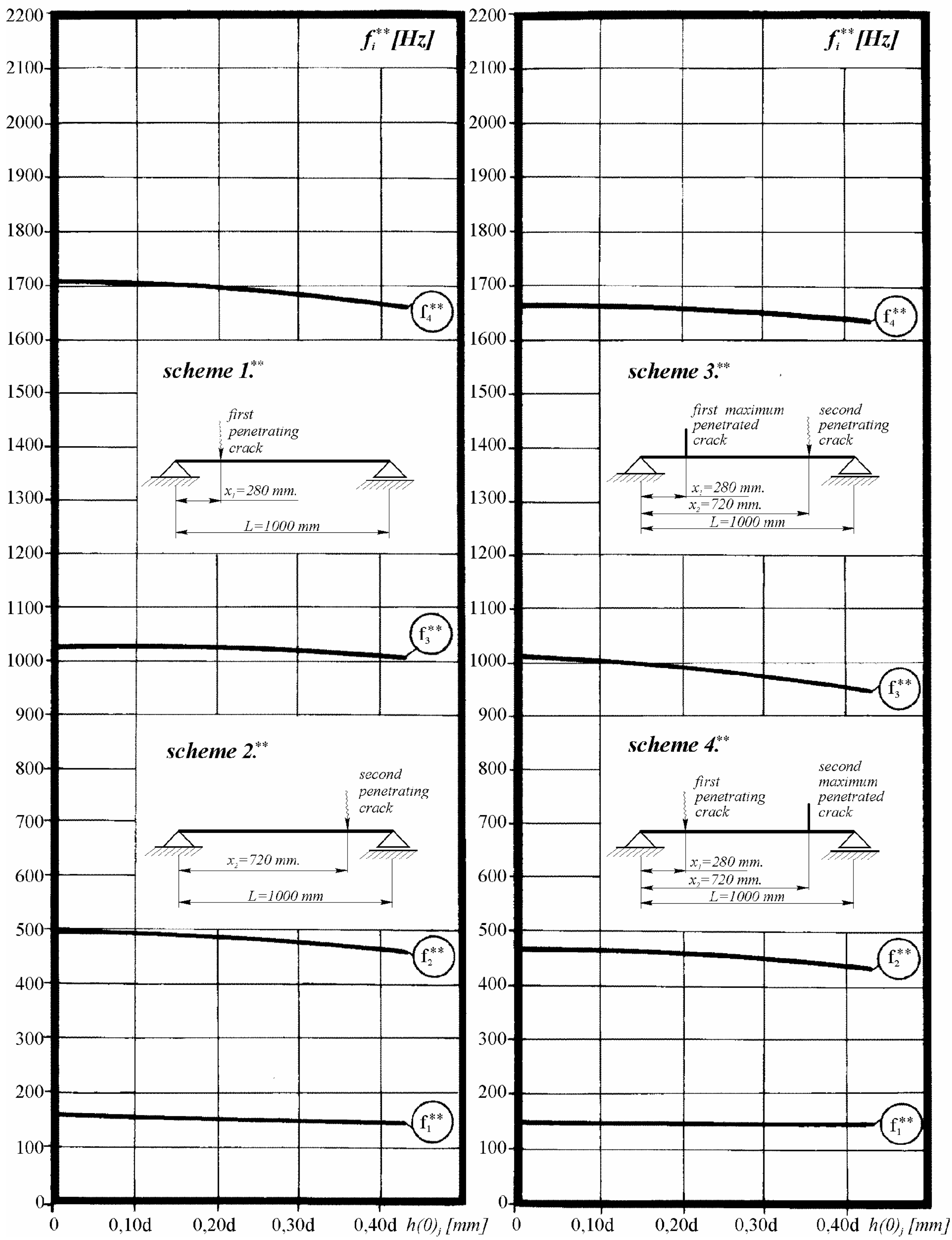

Fig.3. Dependency of the first four own frequencies f_i of the transverse vibrations for a steel elastic shaft with hinged fixing of the border sections on the locality x_j and the depth of penetration $h(0)_j$ of the developing transverse one-side symmetrical crack.**

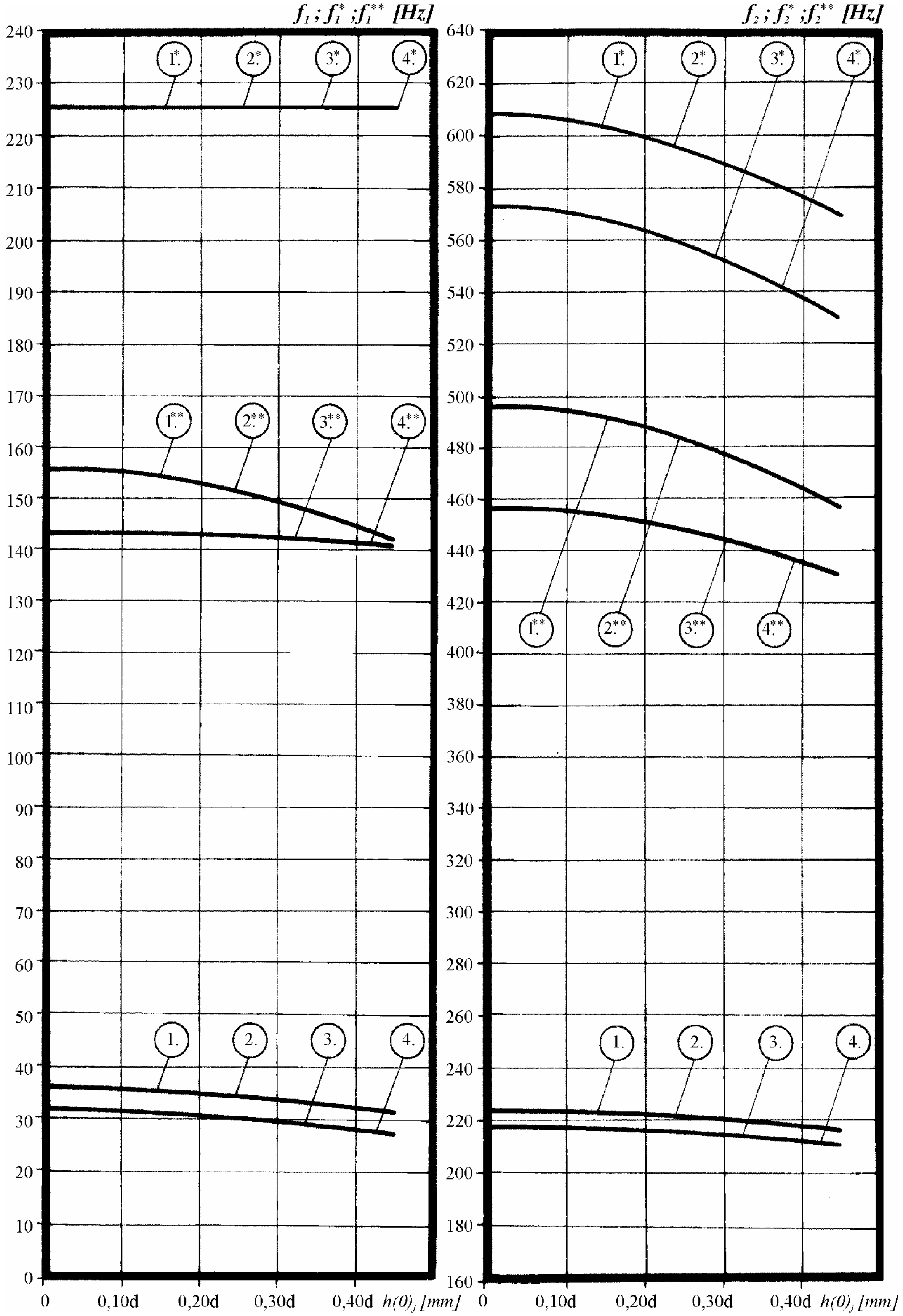

Fig.4. Dependency of the first and second own frequencies of the transverse vibrations of a steel elastic shaft at different fixings of the border sections on the locality x_j and the depth of penetration $h(0)_j$ of the developing transverse one-side symmetrical crack.

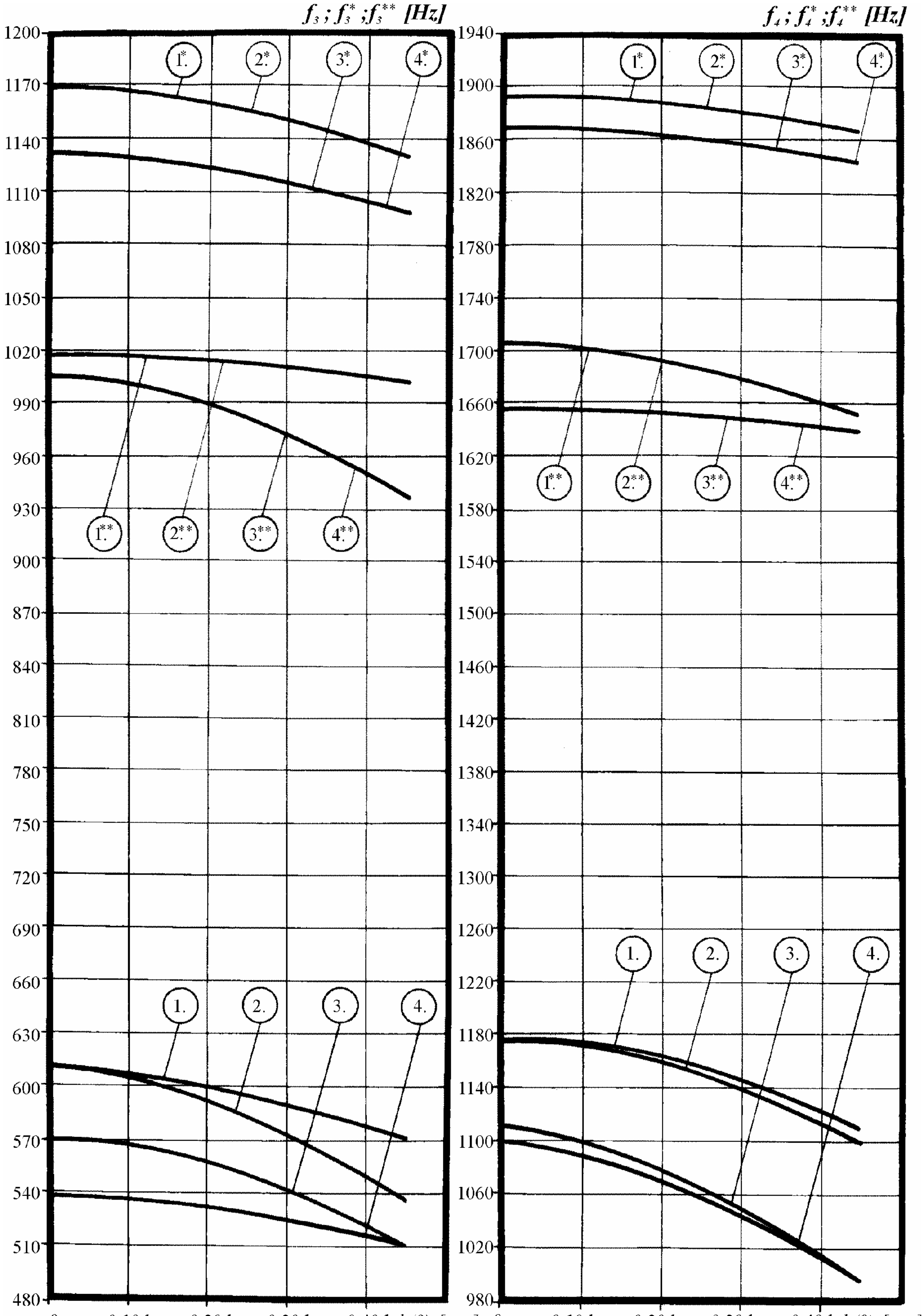

Fig.5. Dependency of the third and fourth own frequencies of the transverse vibrations of a steel elastic shaft at different fixings of the border sections on the locality x_j and the depth of penetration $h(0)_j$ of the developing transverse one-side symmetrical crack.