ON INFLUENCE OF RESIDUAL STRESSES ON FRACTURE OF COMPOSITE MATERIALS CONTAINING INTERACTING CRACKS

Viacheslav BOGDANOV

S.P. Timoshenko Institute of Mechanics, National Academy of Sciences,
Department of Fracture Mechanics, Kyiv, Ukraine

ABSTRACT

Considered in this study are the axis-symmetric problems of fracture of composite materials with interacting cracks, which are subjected to residual (initial) stresses parallel to the cracks planes. An analytical approach in the framework of three-dimensional linearised mechanics of solids is used. Two geometrical schemes of cracks placement are studied: a circular crack is located parallel to the surface of a semi-infinite composite with residual stresses and two parallel penny-shaped cracks are contained in an infinite composite material with residual stresses. The cracks assumed to be under a normal or a radial shear load.

Analysis involves reducing the problems to the systems of Fredholm integral equations of the second kind, where the solutions are identified with harmonic potential functions. The representations of the stress intensity factors near the cracks edges are obtained. These stress intensity factors are influenced by the residual stresses. The presence of the free boundary and the interaction between cracks has significant effects on the stress intensity factors as well.

The parameters of fracture for two types of composites (a laminar composite made of aluminum/boron/silicate glass with epoxymaleinic resin and a carbon/plastic composite with stochastic reinforcement by short ellipsoidal carbon fibers) are analyzed numerically. The dependence of the stress intensity factors on the residual stresses, physical-mechanical parameters of the composites and the geometrical parameters of the problem are investigated.

Key words: composites, residual stresses, circular cracks, stress intensity factors

1. INTRODUCTION

Process of composites making often causes residual (technological) strain and stress as well as defects (cracks, exfoliations) in composite materials. These residual stresses may influence considerably on the cracks propagation in composites [1]. When the residaul stresses are oriented parallel to the crack planes their influence on fracture parameters cannot be modeled in the framework of the concept of linear elastic fracture mechanics with classical fracture criteria Griffith-Irwin type because of missing stress components acting along cracks in the criteria mentioned.

A method of approach for the problem of failure of materials with initial (residual) stresses acting along the crack surfaces was developed by A.N.Guz' [1, 2] on the basis of relations of the three-dimensional linearised solids mechanics. Some static and dynamic problems for isolated cracks in homogeneous infinite solids were solved in [1, 3]. Solutions for a pre-stresses isotropic homogeneous half-space with a penny-shaped crack under normal pressure and radial shear are presented in [4, 5].

The aim of the present work is studying the influence of residual stresses on fracture of a semi-infinite composite with a circular crack and on fracture of a infinite composite containing two parallel disk-shaped cracks. The cracks assumed to be under normal or radial shear loads. It is assumed that dimensions of the cracks are essentially greater that the dimensions of structural elements of the composites, i.e. the macro-cracks are considered. Under the assumptions mentioned the composite material are modeled by an anisotropic solid with reduced mechanical characteristics.

2. PROBLEM FORMULATION

The relationships of the second variant of small initial strain theory [1] are used. The initial state caused by initial (residual) stresses is determined by geometric-linear theory. With the reference to a system of Cartesian coordinates x_j $(j = 1,2,3)$, the components of the stress tensor are given by σ_{ij} and the components of the displacement vector by u_j.

An initial tension (compression) is applied in the Ox_1x_2-plane. This results in a uniform initial stress and strain state

$$\sigma_{33}^0 = 0, \quad \sigma_{11}^0 = \sigma_{22}^0 \neq 0, \quad \sigma_{11}^0 = const,$$
$$u_j^0 = \delta_{jm}\left(\lambda_j - 1\right)x_m; \quad \lambda_1 = \lambda_2 \neq \lambda_3, \quad \lambda_j = const, \tag{1}$$

where λ_j are the extensional (contractional) ratio while δ_{ij} is Kronecker's symbol.

In [1, 3], the general solutions of linearised equations of equilibrium for the uniform initial state in Eq. (1) are obtained in terms of potential functions. These solutions depend on the roots n_1 and n_2 of the governing characteristic equations. For problems with axis-symmetry, a solution for different roots (which is realized for composites materials) is given by

$$u_r = \frac{\partial}{\partial r}(\varphi_1 + \varphi_2) \;;\; u_3 = m_1 n_1^{-1/2}\frac{\partial \varphi_1}{\partial z_1} + m_2 n_2^{-1/2}\frac{\partial \varphi_2}{\partial z_2} \;;$$
$$t_{33} = C_{44}\left(d_1 l_1 \frac{\partial^2 \varphi_1}{\partial z_1^2} + d_2 l_2 \frac{\partial^2 \varphi_2}{\partial z_2^2}\right);\; t_{3r} = C_{44}\frac{\partial}{\partial r}\left[n_1^{-1/2} d_1 \frac{\partial \varphi_1}{\partial z_1} + n_2^{-1/2} d_2 \frac{\partial \varphi_2}{\partial z_2}\right], \tag{2}$$

where (r, θ, x_3) are cylindrical coordinates obtained from Cartesian coordinates x_j $(j = 1,2,3)$, $z_i \equiv n_i^{-1/2} x_3$, $(i = 1,2)$, $\varphi_i(r, z_i)$ are harmonic potential functions. The values $C_{44}, m_i, l_i, n_i, d_i$ $(i = 1,2)$ in Eqs. (3) depend on the initial stresses as far as on the material properties. For linear material model

$$n_{1,2} = \frac{1}{2}\left(\mu_{13} + \sigma_{11}^0\right)^{-1}\left(a_{11} + \sigma_{11}^0\right)^{-1}\Big\{\left(a_{11}a_{33} + \sigma_{11}^0 a_{33} + \sigma_{11}^0 \mu_{13} - 2a_{13}\mu_{13} - a_{13}^2\right) \pm$$
$$\pm\sqrt{\left(a_{11}a_{33} + \sigma_{11}^0 a_{33} + \sigma_{11}^0 \mu_{13} - 2a_{13}\mu_{13} - a_{13}^2\right)^2 - 4\left(a_{11} + \sigma_{11}^0\right)\left(\mu_{13} + \sigma_{11}^0\right)\mu_{13}a_{33}}\Big\} \;;$$

$$C_{44} = \mu_{13};\ m_j = \left[\left(a_{11} + \sigma_{11}^0\right)n_j - \mu_{13}\right]\left(a_{13} + \mu_{13}\right)^{-1};\ d_j = 1 + m_j; \tag{3}$$
$$l_j = \left[n_j\left(a_{11}a_{33} + \sigma_{11}^0 a_{33} - a_{13}^2 - a_{13}\mu_{13}\right) - a_{33}\mu_{13}\right]\left[n_j\left(a_{11} + \sigma_{11}^0\right) + a_{13}\right]^{-1} n_j^{-1}\mu_{13}^{-1};\ j = 1,2.$$

For transversally-isotropic composites parameters a_{ij}, μ_{ij} are not depend on the initial stresses σ_{11}^0 and obtained as

$$a_{11} = E(1-\nu'\nu'')a^{-1};\ a_{33} = E'\left(1-\nu^2\right)a^{-1};\ a_{13} = E\nu'(1+\nu)a^{-1};\ a = 1-\nu^2 - 2\nu'\nu'' - 2\nu\nu'\nu''$$
$$\mu_{12} = G \equiv G_{12} = \frac{1}{2}E(1+\nu)^{-1};\ \mu_{13} = G' \equiv G_{13};\ \nu \equiv \nu_{12};\ \nu' \equiv \nu_{31};\ \nu'' \equiv \nu_{13}. \tag{4}$$

2.1. A near-surface crack in a semi-infinite composite

Let us consider a semi-infinite composite that is bounded by a plane $x_3 = -h$. A penny shaped crack with radius a lies in the upper halfspace $x_3 \geq -h$ in the plane $x_3 = 0$ with the centre on the Ox_3-axes. The origin of the cylindrical coordinates coincides with the center of the crack.

On the faces of the crack, normal stresses $\sigma(r)$ (symmetrical with respect to the plane $x_3 = 0$) and radial stresses $\tau(r)$ (anti-symmetrical with respect to the plane $x_3 = 0$) are specified. The values of these stresses are assumed to be small in comparison with the value of the initial stresses σ_{11}^0. Only axis-symmetric stress and strain distribution considered. The boundary of the half-space is stress-free. The boundary conditions are

$$t_{33} = -\sigma(r)\ ;\ t_{3r} = -\tau(r)\ ;\ \left(x_3 = \pm 0,\quad 0 \leq r < a\right),$$
$$t_{33} = 0\ ;\ t_{3r} = 0\ ;\ \left(x_3 = -h,\quad 0 \leq r < \infty\right). \tag{5}$$

Here t_{ij} are the components of non-symmetric 1st Piola-Kirchhoff stress tensor.

The solid can be divided into two regions: 1- the half-space $x_3 \geq 0$ and 2 – the layer $-h \leq x_3 \leq 0$. At the regions boundary outside the crack ($x_3 = 0, r > a$) the stresses and the displacements should all be continuous. This requires the following additional conditions

$$u_3^{(1)} = u_3^{(2)},\quad u_r^{(1)} = u_r^{(2)},\quad \left(x_3 = 0,\ r \geq a\right), \tag{6}$$
$$t_{33}^{(1)} = t_{33}^{(2)},\quad t_{3r}^{(1)} = t_{3r}^{(2)},\quad \left(x_3 = 0,\ 0 \leq r < \infty\right), \tag{7}$$
$$t_{33}^{(2)} = -\sigma(t),\quad t_{3r}^{(2)} = -\tau(r),\quad \left(x_3 = 0,\ 0 \leq r < a\right), \tag{8}$$
$$t_{33}^{(2)} = 0,\quad t_{3r}^{(2)} = 0,\quad \left(x_3 = -h,\ 0 \leq r < \infty\right). \tag{9}$$

Moreover, the stresses and displacements in the half-space $x_3 \geq 0$ must vanish for large values of x_3.

2.2. Two parallel cracks in an infinite composite

Let us consider two circular cracks with equal radiuses a, which are located in parallel planes $x_3 = 0$ and $x_3 = -2h$ with centers on the axys Ox_3. For this cracks placement there is a symmetry of geometrical and stress-strain schemes of the problem with the plane $x = -h$. Therefore, the problem for the space containing two parallel cracks may be formulated in terms of a problem for a half-space with a near-surface crack. On the faces of the crack, normal stresses $\sigma(r)$ (symmetrical with respect to the plane $x_3 = 0$) and radial stresses $\tau(r)$

(anti-symmetrical with respect to the plane $x_3 = 0$) are specified. Considering the upper half-space $x_3 \geq -h$ we have boundary conditions on the crack faces and on the plane $x = -h$

$$t_{33} = -\sigma(r),\ t_{3r} = -\tau(r) \qquad (x_3 = \pm 0,\ 0 \leq r \leq a)$$
$$u_3 = 0,\ t_{3r} = 0 \qquad (x_3 = -h, 0 \leq r < \infty). \tag{10}$$

Let us divide the solid into two regions: 1- the half-space $x_3 \geq 0$ and 2 – the layer $-h \leq x_3 \leq 0$. At the regions boundary outside the crack ($x_3 = 0, r > a$) the stresses and the displacements should all be continuous. This requires the following additional conditions

$$u_3^{(1)} = u_3^{(2)}, \quad u_r^{(1)} = u_r^{(2)}, \quad (x_3 = 0, r \geq a), \tag{11}$$
$$t_{33}^{(1)} = t_{33}^{(2)}, \quad t_{3r}^{(1)} = t_{3r}^{(2)}, \quad (x_3 = 0, 0 \leq r < \infty), \tag{12}$$
$$t_{33}^{(2)} = -\sigma(t), \quad t_{3r}^{(2)} = -\tau(r), \quad (x_3 = 0, 0 \leq r < a), \tag{13}$$
$$u_3^{(2)} = 0, \quad t_{3r}^{(2)} = 0, \quad (x_3 = -h, 0 \leq r < \infty). \tag{14}$$

3. FREDHOLM INTEGRAL EQUATIONS OF THE SECOND KIND

The problems can be reduced to systems of dual integral equations and then to Fredholm integral equations of the second kind. Referring to regions 1 and 2, the harmonic potential functions φ_1 and φ_2 can be expressed in form of the Hankel integrals

$$\varphi_1^{(1)}(r, z_1) = \int_0^\infty A(\lambda) e^{-\lambda z_1} J_0(\lambda r) \frac{d\lambda}{\lambda};\ \varphi_2^{(1)}(r, z_2) = \int_0^\infty B(\lambda) e^{-\lambda z_2} J_0(\lambda r) \frac{d\lambda}{\lambda};$$
$$\varphi_1^{(2)}(r, z_1) = \int_0^\infty [C_1(\lambda) ch\lambda(z_1 + h_1) + C_2(\lambda) sh\lambda(z_1 + h_1)] J_0(\lambda r) \frac{\partial\lambda}{\lambda sh\lambda h_1}; \tag{15}$$
$$\varphi_2^{(2)}(r, z_2) = \int_0^\infty [D_1(\lambda) ch\lambda(z_2 + h_2) + D_2(\lambda) sh\lambda(z_2 + h_2)] J_0(\lambda r) \frac{\partial\lambda}{\lambda sh\lambda h_2};$$

where $h_i = hn_i^{-1/2}$, $(i = 1,2)$, $J_0(\lambda r)$ denotes the Bessel function. The functions $A(\lambda)$, $B(\lambda)$, $D_j(\lambda)$, $C_j(\lambda)$, $(j = 1,2)$ remain to be found.

3.1. A near-surface crack in a semi-infinite composite

Conditions (7) та (9) specified on the entire plane of $x_3 = \text{const}$ allow the determination of the functions $A(\lambda), B(\lambda), D_j(\lambda)$ in terms of functions $C_j(\lambda)$, $(j = 1,2)$. From the Eqs. (6) and (8) the following system of dual integral equations is obtained

$$\int_0^\infty \left[C_1(\lambda)(cth\mu_1 - \gamma cth\mu_2) + C_2(\lambda)\left(1 - \frac{k_2}{k_1}\gamma\right)\right] J_0(\lambda r)\lambda\, d\lambda = -\frac{\sigma(r)}{C_{44} d_1 l_1} \quad (r < a);$$
$$\int_0^\infty \left[C_1(\lambda)\left(1 - \frac{k_1}{k_2}\gamma\right) + C_2(\lambda)(cth\mu_1 - \gamma cth\mu_2)\right] J_1(\lambda r)\lambda\, d\lambda = \frac{\tau(r)\sqrt{n_1}}{C_{44} d_1} \quad (r < a) \tag{16}$$
$$\int_0^\infty X_1 J_0(\lambda r)\, d\lambda = 0 \quad (r > a);\qquad \int_0^\infty X_2 J_1(\lambda r)\, d\lambda = 0 \quad (r > a),$$

with

$$X_1 = C_1(\lambda)\frac{k_1}{k}\left[(1+cth\mu_1)-\gamma(1+cth\mu_2)\right]+C_2(\lambda)\frac{k_2}{k}\left[\frac{k_1}{k_2}(1+cth\mu_1)-\gamma(1+cth\mu_2)\right] \quad ;$$

$$X_2 = C_1(\lambda)\frac{k_2}{k}\left[(1+cth\mu_1)-\frac{k_1}{k_2}\gamma(1+cth\mu_2)\right]+C_2(\lambda)\frac{k_2}{k}\left[(1+cth\mu_1)-\gamma(1+cth\mu_2)\right]. \quad (17)$$

The derived system of dual integral equations (16) is solved using the procedure, which is analogous to that presented in [6]. We seek the solution of the system of Eqs. (16) in the form

$$X_1 = \int_0^a \varphi(t)\sin\lambda t\, dt\,; \qquad X_2 = \sqrt{\frac{\pi\lambda}{2}}\int_0^a \sqrt{t}\,\psi(t)J_{3/2}(\lambda t)\,dt\,, \quad (18)$$

where $\varphi(t), \psi(t)$ are unknown functions, continuous together with their derivatives in the interval [0, a].

The third and the fourth equations in (16) are automatically satisfied. Functions $C_j(\lambda)$ $(j=1,2)$ are expressible in terms of $X_1(\lambda), X_2(\lambda)$. The substitution of these quantities into the first and second Eqs. (16) for $r < a$ gives the Fredholm integral equations of the second kind, which may be given in the dimensionless form

$$f(\xi)+\frac{4k_1}{\pi k}\int_0^1 f(\eta)K_{11}(\xi,\eta)\,d\eta-\frac{4k_1}{\pi k}\int_0^1 g(\eta)K_{12}(\xi,\eta)\,d\eta=-\frac{4k_1}{\pi k}\int_0^{\pi/2} s(\xi\sin\theta)d\theta\,,$$

$$g(\xi)+\frac{4k_2}{\pi k}\int_0^1 f(\eta)K_{21}(\xi,\eta)\,d\eta-\frac{4k_2}{\pi k}\int_0^1 g(\eta)K_{22}(\xi,\eta)\,d\eta=\frac{4k_2}{\pi k}\xi\int_0^{\pi/2} q'(\xi\sin\theta)\,d\theta\,, \quad (19)$$

where

$$f(\xi)\equiv a^{-1}\varphi(a\xi), \quad g(\xi)\equiv a^{-1}\frac{d}{d\xi}\left[\xi\psi(a\xi)\right], \quad s(\xi)=\frac{\xi t(\xi)}{C_{44}d_1 l_1}\,,\ q(\xi)\equiv\frac{\xi p(\xi)}{C_{44}n_1^{-1/2}d_1}\,,$$

$$t(\xi)\equiv\sigma(a\xi)\,,\ p(\xi)\equiv\tau(a\xi). \quad (20)$$

The kernels of integral equations (19) are

$$K_{11}(\xi,\eta)=\frac{k_2}{k}\left[2I_1(\beta_1+\beta_2,\eta)-\frac{k_1+k_2}{2k_2}I_1(2\beta_1,\eta)-\frac{k_1+k_2}{2k_1}I_1(2\beta_2,\eta)\right] \quad ;$$

$$K_{12}(\xi,\eta)=\frac{k_1+k_2}{k}\{\left[\eta^{-1}I_0(\beta_1+\beta_2,\eta)-I_0(\beta_1+\beta_2,1)\right]-$$
$$\frac{1}{2}\left[\eta^{-1}I_0(2\beta_1,\eta)-I_0(2\beta_1,1)\right]-\frac{1}{2}\left[\eta^{-1}I_0(2\beta_2,\eta)-I_0(2\beta_2,1)\right]\}$$

$$K_{21}(\xi,\eta)=-\frac{k_1+k_2}{k}\xi\left[I_2(\beta_1+\beta_2,\eta)-\frac{1}{2}I_2(2\beta_1,\eta)-\frac{1}{2}I_2(2\beta_2,\eta)\right]\ ; \quad (21)$$

$$K_{22}(\xi,\eta)=-\frac{k_1}{k}\xi\{2\left[\eta^{-1}I_1(\beta_1+\beta_2,\eta)-I_1(\beta_1+\beta_2,1)\right]-\frac{k_1+k_2}{2k_1}\left[\eta^{-1}I_1(2\beta_1,\eta)-I_1(2\beta_1,1)\right]-$$
$$\frac{k_1+k_2}{2k_2}\left[\eta^{-1}I_1(2\beta_2,\eta)-I_1(2\beta_2,1)\right]\},$$

with

$$I_0(\beta,\eta)=\frac{1}{4}\ln\frac{\beta^2+(\xi+\eta)^2}{\beta^2+(\xi-\eta)^2}=\frac{1}{4}\ln\frac{\zeta+1}{\zeta-1};\; I_1(\beta,\eta)=\frac{\beta}{2\xi\eta\left(\zeta^2-1\right)};$$
$$I_2(\beta,\eta)=I_1(\beta,\eta)\left[4\zeta\; I_1(\beta,\eta)-\frac{1}{\beta}\right],$$
$$\text{where}\,\zeta=\frac{\beta^2+\xi^2+\eta^2}{2\xi\eta}\;,\;\beta=ha^{-1}\;,\;\beta_i=\beta n_i^{-1/2}\quad(i=1,2).$$

3.2. Two parallel cracks in an infinite composite

Using the same procedure that in 3.1. it can be obtained the system of the Fredholm integral equations of the second kind in the dimensionless form

$$f(\xi)+\frac{2}{\pi}\int_0^1 f(\eta)K_{11}(\xi,\eta)d\eta+\frac{2}{\pi}\int_0^1 g(\eta)K_{12}(\xi,\eta)d\eta=-\frac{4}{\pi}\int_0^{\pi/2}s(\xi\sin\theta)d\theta\;,$$
$$g(\xi)+\frac{2}{\pi}\int_0^1 f(\eta)K_{21}(\xi,\eta)d\eta+\frac{2}{\pi}\int_0^1 g(\eta)K_{22}(\xi,\eta)d\eta=\frac{4}{\pi}\xi\int q'(\xi\sin\theta)d\theta\;,\tag{22}$$

where

$$s(\xi)=\frac{\xi t(\xi)}{C_{44}d_2l_2},\; q(\xi)\equiv\frac{\xi p(\xi)}{C_{44}n_2^{-1/2}d_2},\; t(\xi)\equiv\sigma(a\xi),\; p(\xi)\equiv\tau(a\xi).$$

The kernels of the integral equations (22) are

$$K_{11}(\xi,\eta)=k_1k^{-1}I_1(2\beta_1,\eta)-k_2k^{-1}I_1(2\beta_2,\eta);$$
$$K_{12}(\xi,\eta)=k_1k^{-1}\left\{\left[I_0(2\beta_1,1)-I_0(2\beta_2,1)\right]-\eta^{-1}\left[I_0(2\beta_1,\eta)-I_0(2\beta_2,\eta)\right]\right\};$$
$$K_{21}(\xi,\eta)=k_2k^{-1}\xi\left[I_2(2\beta_1,\eta)-I_2(2\beta_2,\eta)\right];$$
$$K_{22}(\xi,\eta)=-\xi k^{-1}\left\{\left[k_2I_1(2\beta_1,1)-k_1I_1(2\beta_2,1)\right]-\eta^{-1}\left[k_2I_1(2\beta_1,\eta)-k_1I_1(2\beta_1,\eta)\right]\right\}$$

4. STRESS INTENSITY FACTORS

Similar to the classical case [7] we determine the stress intensity factors as coefficients with singularities in the stress components near the tips of the crack

$$K_I=\lim_{r\to+a}\left[2\pi(r-a)\right]^{-1/2}t_{33}(r,0),\quad K_{II}=\lim_{r\to+a}\left[2\pi(r-a)\right]^{-1/2}t_{3r}(r,0)\tag{23}$$

4.1. A near-surface crack in a semi-infinite composite

System of simultaneous Fredholm integral equations of the second kind (19) may by solved numerically. Than, based on the Eqs (2), (15), (17)-(20) the expressions for components of the stress tensor can be obtained as

$$t_{33}^{(2)}(r,0) = C_{44} d_1 l_1 \{ \frac{k}{2k_1} \int_0^a \varphi(t) dt \int_0^\infty \sin \lambda t \; J_0(\lambda r) \lambda d\lambda +$$

$$\int_0^a \varphi(t) dt \int_0^\infty \left[2\frac{k_2}{k} e^{-\mu_1-\mu_2} - \frac{1}{2}\frac{k_1+k_2}{k}\left(\frac{k_2}{k_1} e^{-2\mu_2} + e^{-2\mu_1} \right)\right] \sin \lambda t \; J_0(\lambda r) \lambda d\lambda -$$

$$\sqrt{\frac{\pi}{2}} \int_0^a \sqrt{t} \varphi \;(t) dt \int_0^\infty \left[\frac{k_1+k_2}{k} e^{-\mu_1-\mu_2} - \frac{1}{2}\frac{k_1+k_2}{k}\left(e^{-2\mu_1} + e^{-2\mu_2} \right)\right] J_{3/2}(\lambda t) J_0(\lambda r) \lambda^{3/2} d\lambda \}$$

$$t_{3r}^{(2)}(r,0) = -C_{44} n_1^{-1/2} d_1 \{ \frac{k}{2k_2} \sqrt{\frac{\pi}{2}} \int_0^a \sqrt{t} \psi(t) dt \int_0^\infty J_{3/2}(\lambda t) J_1(\lambda r) \lambda^{3/2} d\lambda +$$

$$\sqrt{\pi} \int_0^a \sqrt{t} \psi(t) dt \int_0^\infty \left[2\frac{k_1}{k} e^{-\mu_1-\mu_2} - \frac{1}{2}\frac{k_1+k_2}{k}\left(\frac{k_1}{k_2} e^{-2\mu_2} + e^{-2\nu_1} \right)\right] J_{3/2}(\lambda t) J_1(\lambda r) \lambda^{3/2} d\lambda -$$

$$\int_0^a \varphi(t) dt \int_0^\infty \left[\frac{k_1+k_2}{k} e^{-\mu_1-\mu_2} - \frac{1}{2}\frac{k_1+k_2}{k}\left(e^{-2\mu} + e^{-2\mu_2} \right)\right] \sin \lambda t \; J_1(\lambda r) \lambda \; d\lambda \}. \quad (24)$$

Analysis of the Eqs. (24) shows that at $r \to +a$

$$t_{33}^{(2)}(r,0) \approx -C_{44} d_1 l_1 \frac{k}{2k_1} \frac{\varphi(a)}{\sqrt{r^2-a^2}}; \qquad t_{3r}^{(2)}(r,0) \approx C_{44} n_1^{-1/2} d_1 \frac{k}{2k_2} \frac{a\psi(a)}{r\sqrt{r^2-a^2}}. \quad (25)$$

From Eqs. (24), (25) it follows

$$K_I = -C_{44} d_1 l_1 \frac{k}{2k_1} \sqrt{\frac{\pi}{a}} \varphi(a); \; K_{II} = C_{44} n_1^{-1/2} d_1 \frac{k}{2k_2} \sqrt{\frac{\pi}{a}} \psi(a). \quad (26)$$

Using dimensionless variables and functions, it can be obtained

$$K_I = -C_{44} d_1 l_1 \frac{k}{2k_1} \sqrt{\pi a} f(1) \quad ; \; K_{II} = C_{44} n_1^{-1/2} d_1 \frac{k}{2k_2} \sqrt{\pi a} \int_0^1 g(\xi) d\xi , \quad (27)$$

where the functions $f(\xi)$ and $g(\xi)$ should be determined from the system of Fredholm integral equations (19).

4.2. Two parallel cracks in an infinite composite

Similar to 4.1. it may be obtained representations for stress intensity factor for two parallel cracks in an infinite composite

$$K_I = -\frac{1}{2} C_{44} d_2 l_2 \sqrt{\pi a} f(1) \quad ; \; K_{II} = \frac{1}{2} C_{44} n_2^{-1/2} d_2 \sqrt{\pi a} \int_0^1 g(\xi) d\xi , \quad (28)$$

where the functions $f(\xi)$ and $g(\xi)$ should be determined from the system of Fredholm integral equations (22).

It follows from Eqs. (27), (28) that the presence of the free boundary and interaction of two parallel cracks lead to nontrivial stress intensity factors K_{II} for cracks under normal rupture (when radial shear is equal to zero) (for crack in infinite solid $K_{II} = 0$ [1]). On the other hand for a near-surface crack under radial shear and zero normal stress as far as for two parallel cracks under radial shear and zero normal stress the stress intensity factors K_I are nontrivial (for crack in infinite solid $K_I = 0$ [1]). Meanwhile, both of the stress intensity factors K_{I} and K_{II} are effected by the initial stress $\sigma_{11}^0 = \sigma_{22}^0$ (or extension ratio $\lambda_1 = \lambda_2$) and also depended on the distance *h* (or β) from the crack to the free boundary, since the solutions $f(\xi)$ and $g(\xi)$ of Eqs. (19) and (22) depend on these parameters.

5. NUMERICAL RESULTS

The Bubnov-Galerkin method has been used in the numerical analysis of the system of Fredholm integral equations of the second kind (19) and (22). Gaussian-quadrature formulae were utilized for numerical integration. Results are given *for the case of a near-surface crack under uniform loads* $\sigma(t) = \sigma_0$ and $\tau(r) = \tau_0$.

Below there are presented numerical results for case when composites can be modeled by transversally-isotropic materials with values $C_{44}, n_i, l_i, m_i, d_i, k_1, k_2, k$ $(i = 1,2)$, which may be obtained from Eqs. (4), (5).

5.1.A laminar composite with isotropic layers.

The crack is located in plane $x_3 = 0$ that is parallel to the layers of the composite and free boundary of the solid. The mechanical macro-characteristics of this composite material may be obtained by the elastic characteristics of its components and their volume concentration in the composite [8]. The numerical results for a laminar composite, made of aluminum/boron/silicate glass with an epoxy-maleinic resin are obtained.

Consider the example of a crack subjected to uniform normal stress $\sigma(t) = \sigma_0$, $\tau(r) = 0$. Variations of the stress intensity factor ratio K_I / K_I^∞ (here K_I^∞ is the stress intensity factor for an isolated Mode I crack in an infinite composite) with the glass concentration factor c_1 are displayed in Fig. 1. Solid lines are for $\beta = h/a = 0.25$, dashed – for $\beta = 0.5$. The lines 1 and 1’ are for $\lambda_1 = 0.99$ (compressive initial (residual) stresses), 2, 2’ – for $\lambda_1 = 1.0$ (initial stresses are absent), 3, 3’ – for $\lambda_1 = 1.1$ (tensile initial (residual) stresses). It is seen that the stress intensity factor ratio depends greatly on the glass concentration factor c_1 as far as on values of initial (residual) elongation (or reduction) ratio λ_1. Besides, for the values of dimensionless distance from the crack to the edge of the composite $\beta = 0.25$ the values of K_{II} / K_{II}^∞ and K_I / K_{II}^∞ are higher than for $\beta = 0.5$.

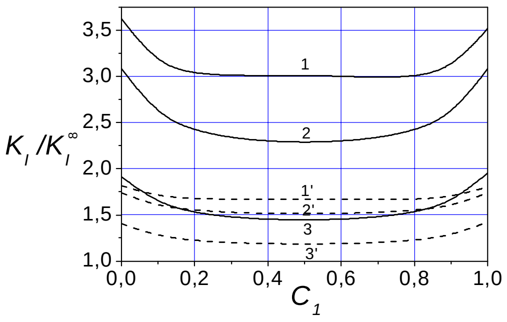

Figure 1.

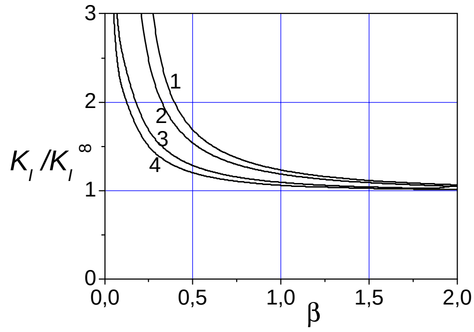

Figure 2.

In Fig. 2, the dependences of the stress intensity factor ratio K_I / K_I^∞ with the dimensionless distance from the crack to the free boundary of the composite β are given. The curves 1, 2, 3 and 4 corresponds to $\lambda_1 = 0.99, 1.0, 1.05$ and 1.1, respectively. The figure shows that the values of stress intensity factor K_I for small values of β are greater than stress intensity factor K_I^∞ for the isolated crack in the infinity composite.

5.2. A composite with stochastic reinforcement in the plane $x_3 = const$ by short elliptical fibers of finite length

In macro-volumes this composite may be modeled as a transversally-isotropic medium [8]. Below, the numerical results are given for a composite that consist of a carbon plastic with stochastic reinforcement by short ellipsoidal carbon fiber (the fiber concentration is 0.7 and the fiber aspect ratio is 10).

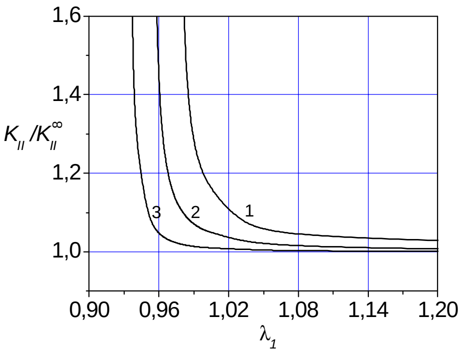

Figure 3.

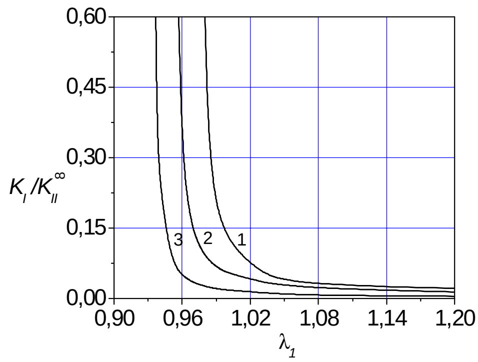

Figure 4.

For a Mode II crack subjected to uniform radial shear stress $\tau(r) = \tau_0$ and zero normal stress $\sigma(t) = 0$) the dependences of the stress intensity factor ratio K_{II} / K_{II}^∞ and K_I / K_{II}^∞ with the initial (residual) elongation (or reduction) ratio λ_1 are displayed in Figs. 3, 4, respectively, for values of $\beta = 0.25$ (curves *1*), $\beta = 0.5$ (curves *2*) and $\beta = 1.0$ (curves *3*).

The curves have vertical asymptotes corresponding to the values of the critical reduction λ_1 obtained in the problem of local instability of the semi-infinite composite material with a circular crack in compression oriented parallel to the crack plane [9].

6. CONCLUSIONS

The obtained results allow making the following conclusions: (1) the presence of a free boundary in a composite with initial stress and interaction of two parallel cracks leads to a nontrivial stress intensity factors K_{II} for cracks under normal stress and nontrivial K_I for cracks under radial shear; (2) the values of stress intensity factors increase with decrease of distance between the crack and the half-space boundary β . As the relative distance β tends to infinity the stress intensity factors tend to the values obtained for an isolated circular crack in an infinite solid with initial stress; (3) the parameters of the composites material influenced greatly on the values of stress intensity factors; (4) the values of stress intensity factors increase abruptly when the initial reduction ratio λ_1 tends to the value, with which there is a local loss of stability of a semi-infinite composite containing a circular crack under compression along the crack plane.

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