LONG-TERM CRACK GROWTH IN 3-D REINFORCED POLYMER

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ABSTRACT

Reinforced polymers offer significant improvement over the pure polymers in mechanical and crack-resistance properties. Investigation of crack propagation under the time-dependent loading in these materials basing on a theory of long-term crack growth in viscoelastic medium requires representation of expression for crack-opening displacement in the form of convolution integral. In this paper rheological characteristics of displacement are obtained for the disk-shaped crack in 3-D reinforced composite with polyethylene filler. As this polymer has sufficiently great difference in instantaneous and sustained stresses an efficient method of error alignment for the approximation in operator domain is applied. For used problem formulation method of operator continued fractions and Laplace transformation method which can also be efficiently applied are equivalent. The propagation of a crack with considerable craze zone is investigated basing on the obtained kernel of convolution integral. Used in this paper approach for finding of rheological characteristics of composite material can also be utilized in the analysis of long-term deformation and crack propagation in different reinforced polymers.

Keywords: viscoelastic composite, viscoelastic properties, crack growth, equations of crack growth

1. INTRODUCTION

Data of experimental researches [1] show stable crack propagation under low values of strength in viscoelastic composite materials. For theoretical research of subcritical crack propagation in [1, 2] the theory of subcritical crack growth in anisotropic viscoelastic medium was proposed. In cited works acceptable agreement of solutions with experimental researches of long-term fracture was shown. Efficiency of the proposed theory was demonstrated by solving of the wide range of new problems on the long-term fracture for both polymeric composites materials and other viscoelastic materials such as concrete, wood and rocks.

Problem solving by means of proposed methodology can be formally divided into three stages. First stage is determination of effective viscoelastic moduli of the composite. The possibility to obtain convenient form of material function for the further solving of boundary-value problem (second stage) extensively depends on chosen kernel for creep function describing deforming of composite constituents. In papers [3, 4] the methodology for construction of kernel in the form of sum of fractional exponential Rabotov’s functions is developed. Mentioned representation makes possible to optimize the set of viscoelastic parameters. If the number of items in the obtained kernel is low effective numeric solution for a problem of composite deforming becomes possible [3]. Number of items increases numeric realization of proposed methodology becomes complicated. It is often happens if exponent is used as a material function component. Micromechanics methods have been used with great success to predict effective moduli of elastic composite materials, given the properties of the components and the volume fractions. Method of operator continued fractions or method of Laplace transform enables finding of corresponding viscoelastic effective moduli [5].

Solution of the problem on long-term deforming of homogenized composite material near the crack tip is the second stage for the investigation of subcritical crack propagation. Precision of the solution on this stage has a significant influence on the service-life value as a result of the solution of crack growth equations (third stage). Deformation near a crack tip is defined by geometric parameters and effective elastic moduli of the composite. Basing on the same principles as on the first stage viscoelastic crack opening displacement can be obtained using solution for a crack in elastic anisotropic medium. The questions on the solving of a boundary-value problem for linear viscoelasticity were discussed in [3-6]. Basing on the constructed methodology the problems of deforming for composites of different reinforcements and viscoelastic properties were solved [3-9].

In the report investigation of the long-term propagation of a mode I penny-shaped crack in 3D armored composite with viscoelastic matrix is presented. For the numeric solutions material function of polyethylene (material of matrix) with a single Rabotnov's function is used.

2. PROBLEM FORMULATION

Consider composite with regular one-type armored elements which representative sizes are small in comparison with the sizes of a body. Fibers are located both in the plane of isotropy along the sides of equilateral triangle and in the line of a perpendicular to the plane of isotropy. The number of stochastically ordered fibers in each direction is assumed to be sufficiently great. Under some conditions [10] deforming of such a composite can be investigated on the macrolevel that makes possible to consider heterogeneous composite material as a homogeneous body with effective mechanical properties. Fiber orientation allows modeling of composite material by transversally isotropic medium. Viscoelastic properties of investigated composite are conditioned by linear viscoelastic properties of the matrix. Both composite constituents are isotropic materials which deforming can be defined by Yung's modulus (E_1 for fibers and E_2^* for matrix) and Poisson's ratios (n_1 for fibers and n_2^* for matrix). Let us note that methodology used in the report allows investigation of composite when both fibers and matrix are viscoelastic and anisotropic [4, 7]. Thus, according to described formulation

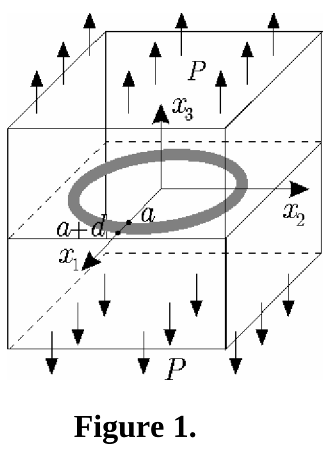


Figure 1.

the composite is modeled by transversally isotropic, homogeneous linearly viscoelastic medium.

To investigate long-term crack propagation by means of the theory of subcritical crack growth in anisotropic viscoelastic medium [1, 2] the modified Dugdale's model with a constant width ring-shaped failure zone will be used. The following assumptions are made: 1) the crack is located in the plane of isotropy; 2) the crack grows in a self-similar fashion in the shape of a penny; 3) the failure zone is modeled by a ring-shaped cut. The cut is located in front of a moving crack subjected to self equilibrating stresses that may be regarded as a modified version of the Dugdale's model; 4) the width of yield zone remains constant (δ = const) [2]. In the report the size of yield zone is assumed to be large in comparison with crack radius that the local field near the crack tip can not be represented by the stress intensity factor.

A uniform external load is applied perpendicularly to the crack plane. Let us note that using applied methodology a various types of time-dependent quasi-static loads can be under consideration [8].

3. CRACK OPENING DISPLACEMENT

When a plane of a penny-shaped crack coincides with isotropy plane (Fig. 1) of transversally isotropic body and loading is applied transversely to the isotropy plane viscoelastic opening displacement of the crack can be written as

$$\delta(r,t) = \tilde{\Lambda}^{*}\delta_0\left[p(t), r, a(t)\right], \quad \tilde{\Lambda}^{*} = \Lambda^{*}/\Lambda, \tag{1}$$

where

$$\delta_0(p,r,a) = \Lambda\, a_0\, p\, \zeta(r/a_0, a/a_0) \tag{2}$$

is elastic crack opening displacement (a_0 and a are the initial and current crack radius correspondingly) at point r; p is load intensity; function $\zeta(\xi,\eta)$ is given by [11]

$$\zeta(\xi,\eta) = \frac{4}{\pi}\left(\frac{\eta}{\rho}+1\right)\left(2\frac{\eta}{\rho}+1\right)^{-1/2} \int\limits_{\arcsin\frac{\eta}{\eta+\rho}}^{\arcsin\frac{\eta}{\xi}} \sqrt{\eta^2 - \xi^2\sin^2\alpha}\,\mathrm{d}\alpha, \tag{3}$$

for $0 < \eta \le \xi \le \eta + \rho$; $\rho = \delta/a_0$ is dimensionless width of a ring-shaped failure zone. If the zone size of nonlinear strains near the crack tip is much smaller than the crack radius a and other characteristic lengths, the local field near the yield zone can be represented by the stress intensity factor. For the crack with small yield zone ($\rho \ll 1$) in eq. (3)

$$\zeta(\xi,\eta) = \frac{\sqrt{8\rho\eta}}{\pi} F\left(\frac{\xi-\eta}{\rho}\right), \quad F(s) = \sqrt{1-s} + \frac{s}{2}\ln\frac{1-\sqrt{1-s}}{1+\sqrt{1-s}}. \tag{4}$$

In eq. (2) Λ is a function of effective properties of transversally isotropic material

$$\Lambda = \frac{\lambda_{11}}{\Delta_{11}}\left(\sqrt{\alpha_2} + \sqrt{\alpha_3}\right), \quad \Delta_{11} = \lambda_{11}\lambda_{33} - \lambda_{13}^2, \tag{5}$$

where α_2 and α_3 are parameters determined by equation [12]

$$\lambda_{11}\lambda_{44}\nu^2 + \left[(\lambda_{13} + \lambda_{44})^2 - \lambda_{11}\lambda_{33} - \lambda_{44}^2\right]\nu + \lambda_{33}\lambda_{44} = 0. \tag{6}$$

In eqs. (5) and (6) l_{ij} are effective moduli of elastic transversally isotropic material.

To predict elastic mechanical characteristics for the investigated composite results of [10] are used in the form

$$l_{ij} = l_{ij}(E_1, n_1, E_2, n_2, c_1, c_{14}), \tag{7}$$

(moduli are not cited here because of theirs complexity), where c_1 is total volume fraction of fibers and c_{14} is along the line of perpendicular to isotropy plane.

As was mentioned in the problem formulation the case of isotropic material of fibers with moduli E_1, n_1 and linear viscoelastic material of matrix with moduli E_2^*, n_2^* is under consideration. Moduli marked with asterisk are operators of linear viscoelasticity:

$$E^* \times f(t) = E_0\left(f(t) + Ò_0^t R(t - t)f(t)\mathrm{d}t\right). \tag{8}$$

Assuming that the cubic strain of viscoelastic component is elastic rheological properties can be reduced to only one characteristic E_2^*. Then effective moduli depending on time can be defined with help of correspondence principle [5] or Voltera's principle using corresponding elastic solution (7) in the form [3]

$$l_{ij}^* = F_{ij}(K^*), \tag{9}$$

where $K^* = E_2^*$.

In modeling of the composite by homogeneous viscoelastic medium due to the representation in eq. (9) viscoelastic moduli can be regarded as functions of elastic characteristics of constituents and a single operator. Thus effective application of operator continued fraction method is possible [3].

Using eqs. (5) and (6) elastic characteristic of opening displacement L can be obtained as

$$L = \frac{l_{11}}{D_{11}}\sqrt{\frac{D_{11} - 2l_{13}l_{44}}{l_{11}l_{44}} + 2\sqrt{\frac{l_{33}}{l_{11}}}}. \tag{10}$$

To obtain this characteristic in time domain use is made of Volterra's principle. According to the principle elastic moduli l_{ij} were replaced by corresponding viscoelastic ones (l_{ij}^*), which were obtained by means of substitution E_2 for its operator E_2^*. Obtained operator function L^* was reduced to the standard form of material function [3].

Let d_c^* stand for the critical opening displacement (COD) that corresponds to the onset of crack growth. Criterion of critical crack opening displacement [1] can be written as

$$d(x,t)\,|_{x=a(t)} = d_c^*. \tag{11}$$

The satisfaction of eq. (11) corresponds to the initiation of stable crack growth. Application of this criterion to crack growth in polymeric materials and using the concept d = const has been cited in [1]. Applying the dimensionless parameter $h_c = a_*/a_0$ (a_0 and a_* are initial and critical crack radiuses) and using eq. (3), the critical COD can be written as

$$\delta_c^* = L\, a_0\, p_0\, \zeta(\eta_c), \tag{12}$$

where $\zeta(\eta) = \zeta(\eta,\eta)$, and function $\zeta(x,\eta)$ is determined by eq. (3) (for the small width of craze zone eq. (4) can be used).

4. SUBCRITICAL CRACK GROWTH

Stable crack growth in a viscoelastic media involves several stages, namely incubation, transitional and stable quasi-static growth. Quasi-static growth passes into dynamic fracture which lasts insignificant (in comparison with mentioned stages) interval [1] and can be negligible in the service life of a body with a crack. According to the theory of subcritical crack growth in viscoelastic medium during all periods of growth time dependence for crack radius is determined implicitly by integral equations

$$\delta(p(t),a_0) + \int_0^{t_*} R(t_* - \tau)\,\delta(p(\tau),a_0)\,\mathrm{d}\tau = \delta_c^*, \tag{14}$$

$$\begin{aligned} &\delta[p(t),a(t)] + \int_0^{t_*} R(t-\tau)\,\delta(p(\tau),a(t),a_0)\,\mathrm{d}\tau + \\ &+ \int_{t_*}^{t} R(t-\tau)\,\delta[p(\tau),a(t),a(\tau)]\,\mathrm{d}\tau = \delta_c^*, \end{aligned} \tag{15}$$

$$\delta[p(t),a(t)] + \int_{t'}^{t} R(t-\tau)\,\delta[p(\tau),a(t),a(\tau)]\,\mathrm{d}\tau = \delta_c^*, \tag{16}$$

where $\delta(p,\eta) \equiv \delta(p,\eta,\eta)$ and t' is solution of the equation $a(t) - a(t') = \delta$.

Solving eqs. (14), (15) and (16) sequentially dependence $a = a(t)$ can be found. Service-life of the viscoelastic composite with a crack can be determined as the end of quasi-static growth.

For material with bounded creep solution of eq. (14) exists only for $\eta < \eta_c^s$, for $\eta \geq \eta_c^s$ value $\delta(t)$ can not reach δ_c^* even for arbitrary large time and crack does not grow. Values of safe crack opening displacement can be found in the form

$$\eta_c^s = \zeta^{-1}\left[\hat{E}_\infty \cdot \zeta(1)\right], \tag{17}$$

where $\hat{E}_\infty = \lim\limits_{t\to\infty} \dfrac{L^* \cdot 1}{L}$.

5. RESULTS

According to the used kernel type in eqs. (14)-(16)

$$R(\theta) = \sum_{i=1}^{m} \lambda_i\, \theta^{-\alpha} \sum_{n=0}^{\infty} \frac{\beta_i^n \theta^{n(1-\alpha)}}{\Gamma[(n+1)(1-\alpha)]}.$$

where α, λ and β are parameters of viscoelastic operator L^* ($0 \leq \alpha < 1, \lambda > 0, \beta < 0$), Γ is Euler’s gamma-function.

As a numerical example crack growth in composite with glass fibers (E = 68671 MPa, n = 0.2) and polyethylene matrix (a = 0,67 , l = 0,18 hour^{a-1} , b = - 0,1745 hour^{a-1} , E = 3600 MPa, n = 0,35) is investigated.

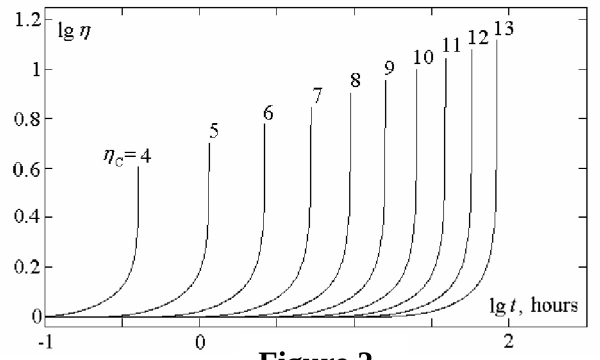
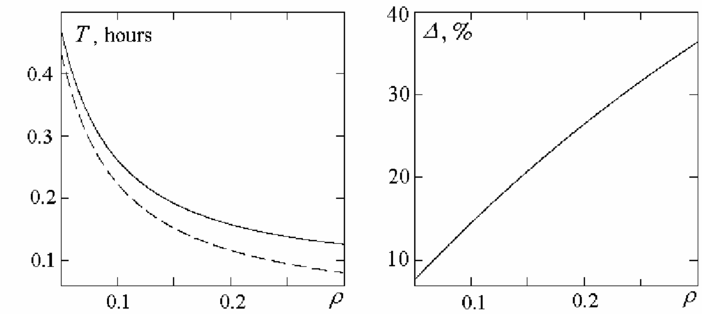


Figure 2.



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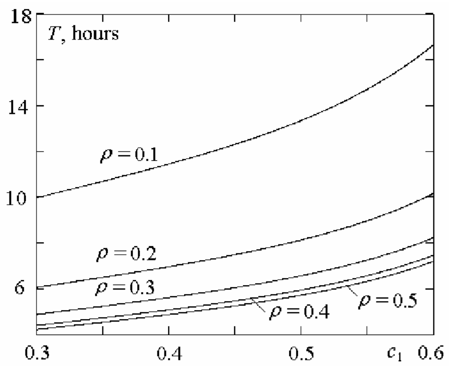

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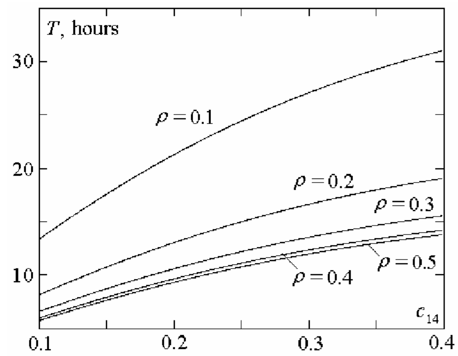

Figure 5.

On Fig. 2 curves of crack growth for $c_1 = 0.45$, $c_{14} = 0.2$, $r = 0.3$ and noted parameters of crack resistance h_c are illustrated. Fig. 3 shows difference in service-life which obtained with help of exact solution for the crack opening displacement (3) (solid line) and approximate one (4) (intermittent line). Second block of the figure shows relative error caused by using of approximation (4) instead of exact solution (3). On the Fig. 4 service-life as a function of volume fraction c_1 for constant value $c_{14} = 0.1$ (so increasing of c_1 gives rise to volume

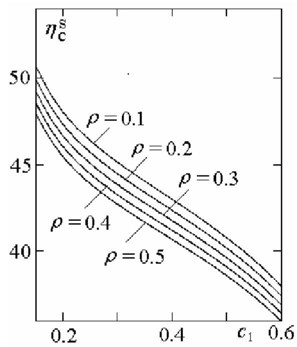

Figure 6.

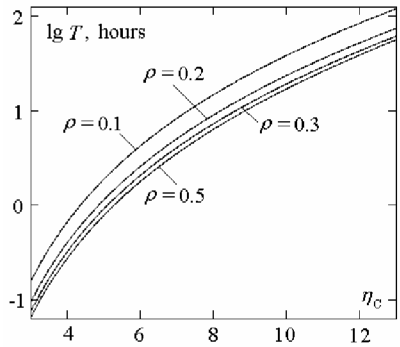

Figure 7.

fraction c_1 - c_{14} of fibers in the isotropy plane) and $h_c = 8$. On Fig. 5 service-life as a function of volume fraction c_{14} is shown for the constant values $c_1 = 0.5$ and $h_c = 8$. Fig. 6 sows how safe crack radius depends on volume fraction c_1 under constant value $c_{14} = 0.15$. On Fig. 7 service-life as a function of crack-resistance parameter h_c is shown.

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