ASYMPTOTICAL SOLVING BEHAVIOR IN FRACTURE MECHANICS PROBLEM FOR NEAR-SURFACE CRACK UNDER INITIAL STRESSES

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ABSTRACT

This paper presents investigation of the asymptotical solving behavior for nonclassical fracture mechanics axisymmetrical problem for a near-surface crack. The crack is located in a near-surface layer parallel to the surface of a halfspace which is subjected to the uniform compression or extension along crack. Asymptotical behavior of stress intensity factors and stresses on a line of a crack is investigated in case distance between a crack and the free boundary approach to infinity or zero.

Key words: Compression, halfspace, interface crack, near-surface crack, stress intensity factors.

1. INTRODUCTION

We will consider a semiinfinite solid containing a penny-shaped crack of radius a which is situated in the upper halfspace $x_3 \geq -h$, in the plane $x_3 = 0$ with centre on Ox_3-axis

The initial stresses that operate along a crack correspond to biaxial uniform compression or extension is defined by [2]:

$$S_{33}^0 = 0, \qquad S_{11}^0 = S_{22}^0 \neq 0,$$
$$u_m^0 = \delta_{jm}(\lambda_j - 1)x_j; \;\; \lambda_1 = \lambda_2 \neq \lambda_3, \;\; \lambda_j = const. \tag{1}$$

Here values λ_j denote the extensional ratio along the axis, x_j - the Lagrangian coordinates conterminous in the undeformed state with cartesian, S_{ij}^0 are the components of the symmetric stress tensor in the initial state, u_j^0 are components of the displacement vector corresponding to the initial stress, δ_{ij} is the Kronecker's symbol.

On crack faces operate small enough relatively with S_{11}^0 normal distributed stress $\sigma(r)$, that is equal and is opposite directional on the upper and lower crack faces.

The axial-symmetric linearized problem, has the following boundary conditions on crack faces $x_3 = \pm 0$ and a free surface $x_3 = -h$ [1]:

$$\begin{aligned} &t_{33} = \sigma(r), \quad t_{3r} = 0 \qquad (x_3 = \pm 0, \ 0 \le r < a); \\ &t_{33} = 0, \qquad t_{3r} = 0 \qquad (x_3 = -h, \ 0 \le r < \infty) \end{aligned} \tag{2}$$

where t_{ij} are Kirchhoff nonsymmetric stress tensor, r, θ, x_3 are cylindrical coordinates obtained from Cartesian coordinates x_j.

The common decisions of the linearized equations for different roots ($n_1^0 \ne n_2^0$) in axial-symmetric case are representable in the shape [2]

$$\begin{aligned} &u_r = \frac{\partial}{\partial r}(\phi_1 + \phi_2), \\ &u_3 = m_1^0 (n_1^0)^{-1/2} \frac{\partial \phi_1}{\partial z_1} + m_2^0 (n_2^0)^{-1/2} \frac{\partial \phi_2}{\partial z_2}, \\ &t_{33} = C_{44}^0 \left(d_1^0 l_1^0 \frac{\partial^2 \phi_1}{\partial z_1^2} + d_2^0 l_2^0 \frac{\partial^2 \phi_2}{\partial z_2^2}\right), \\ &t_{3r} = C_{44}^0 \frac{\partial}{\partial r}\left[(n_1^0)^{-1/2} d_1^0 \frac{\partial \phi_1}{\partial z_1} + (n_2^0)^{-1/2} d_2^0 \frac{\partial \phi_2}{\partial z_2}\right]; \end{aligned} \tag{3}$$

Where $z_i = (n_i^0)^{-1/2} x_3$, $\phi_i(r, z_i)$ - potential harmonic functions

2 FREDHOLM'S INTEGRAL EQUATIONS

The procedure of deriving Fredholm equations is analogous [3] taking into account non zero right part [4]

The half-space $x_3 \ge -h$ is divided into fields "1" $x_3 \ge 0$ and "2" $-h \le x_3 \le 0$ and is used Henkel's integral transformations in these fields.

$$\begin{aligned} &\varphi_1^{(1)}(r, z_1) = \int_0^\infty A(\lambda) e^{-\lambda z_1} J_0(\lambda r) \frac{d\lambda}{\lambda}; \quad \varphi_2^{(1)}(r, z_2) = \int_0^\infty B(\lambda) e^{-\lambda z_2} J_0(\lambda r) \frac{d\lambda}{\lambda}; \\ &\varphi_1^{(2)}(r, z_1) = \int_0^\infty [C_1(\lambda) \mathrm{ch}\lambda(z_1 + h_1) + C_2(\lambda) \mathrm{sh}\lambda(z_1 + h_1)] J_0(\lambda r) \frac{d\lambda}{\lambda \mathrm{sh}\lambda h_1}; \\ &\varphi_2^{(2)}(r, z_2) = \int_0^\infty [D_1(\lambda) \mathrm{ch}\lambda(z_2 + h_2) + D_2(\lambda) \mathrm{sh}\lambda(z_2 + h_2)] J_0(\lambda r) \frac{d\lambda}{\lambda \mathrm{sh}\lambda h_2}; \end{aligned} \tag{4}$$

where $h_i = (n_i^0)^{-1/2} h$

Using a procedure from [3,4] was obtained the equations:

$$\int_0^\infty [d_1 l_1^0 (C_1 \operatorname{cth}\mu_1 + C_2) + d_2 l_2^0 (D_1 \operatorname{cth}\mu_2 + D_2) J_0(\lambda r)\lambda \mathrm{d}\lambda = \frac{\sigma(r)}{c_{44}^0} \qquad (r < a);$$

$$\int_0^\infty [(n_1^0)^{-1/2} d_1 (C_1 + C_2 \operatorname{cth}\mu_1) + (n_2^0)^{-1/2} d_2 (D_1 + D_2 \operatorname{cth}\mu_2) J_0(\lambda r)\mathrm{d}\lambda = \delta \quad (r < a);$$

$$\int_0^\infty x_1 J_0(\lambda r)\mathrm{d}\lambda = 0 \qquad (r > a); \tag{5}$$

$$\int_0^\infty x_2 J_0(\lambda r)\lambda \mathrm{d}\lambda = 0 \quad (r > a);$$

In (5) $\delta = \text{const}$ has dimensionality of length, and unknown functions x, y was coupled by equations:

$$D_1 = -\frac{d_1 l_1^0}{d_2 l_2^0}\frac{\operatorname{sh}\lambda h_2}{\operatorname{sh}\lambda h_1} C_1; \qquad D_2 = -\frac{(n_1^0)^{-1/2} d_1}{(n_2^0)^{-1/2} d_2}\frac{\operatorname{sh}\lambda h_2}{\operatorname{sh}\lambda h_1} C_2; \tag{6}$$

and

$$x_1 = C_1[\frac{k_1}{k}(\operatorname{cth}\mu_1 - \gamma \operatorname{cth}\mu_2) + \frac{k_1}{k}(1-\gamma)] + C_2[\frac{k_1}{k}\operatorname{cth}\mu_1 - \frac{k_2}{k}\gamma \operatorname{cth}\mu_2 + \frac{k_1}{k} - \frac{k_2}{k}\gamma];$$

$$x_2 = C_1[\frac{k_2}{k}\operatorname{cth}\mu_1 - \frac{k_1}{k}\gamma \operatorname{cth}\mu_2 + \frac{k_2}{k} - \frac{k_1}{k}\gamma] + C_2[\frac{k_2}{k}(\operatorname{cth}\mu_1 - \gamma \operatorname{cth}\mu_2) + \frac{k_2}{k}(1-\gamma)]; \tag{7}$$

$$\gamma = \operatorname{sh}\mu_2 / \operatorname{sh}\mu_1; \qquad \mu_i = \lambda h_i, \quad i = 1, 2;$$

$$k_1 = l_1^0 (n_1^0)^{-1/2}, \quad k_2 = l_2^0 (n_2^0)^{-1/2}, \quad k = k_1 - k_2.$$

The solution is selected as conjugate integral equations

$$x_1 = \frac{1}{\lambda}\int_0^a \varphi(t)(\cos\lambda t - \cos\lambda a)dt; \quad x_2 = \frac{1}{\lambda}\int_0^a \psi(t)\cos\lambda t dt, \tag{8}$$

after some transformations [3,4] the system of two integral equations of Fredholm with additional clause has been obtained:

$$f(\xi) + \frac{k_1}{\pi k}\int_0^1 M_1(\xi,\eta) f(\eta) d\eta - \frac{2k_1}{\pi k}\int_0^1 N_1(\xi,\eta) g(\eta) d\eta = \frac{k_1}{\pi k} q(\xi);$$

$$g(\xi) + \frac{k_2}{\pi k}\int_0^1 M_2(\xi,\eta) g(\eta) d\eta - \frac{2k_2}{\pi k}\int_0^1 N_2(\xi,\eta) f(\eta) d\eta + C_1 \frac{k_2}{\pi k} = 0;$$

$$\int_0^1 g(\xi) d\xi = 0 \qquad (0 \le \xi \le 1, 0 \le \eta \le 1); \tag{9}$$

$$f(\xi) \equiv \varphi(a\xi), \quad g(\xi) \equiv \psi(a\xi);$$

$$q(\xi) = 4[p(0) + \xi \int_0^{\pi/2} p'(\xi \sin\zeta) d\zeta]; \quad p(\zeta) \equiv \frac{\sigma(a\zeta)}{C_{44}^0 d_1^0 l_1^0}.$$

Equations (9) are given in dimensionless form. Unknown constant C_1 is attached to the additional condition.

The kernels of the integral equations are defined as

$$M_1(\xi,\eta) = R_1(\eta+\xi) - R_1(1+\xi) + R_1(\eta-\xi) - R_1(1-\xi);$$
$$N_1(\xi,\eta) = S_1(\eta+\xi) + S_1(\eta-\xi); \qquad M_2(\xi,\eta) = S_2(\eta+\xi) + S_2(\eta-\xi);$$
$$N_2(\xi,\eta) = R_2(\eta+\xi) - R_2(1+\xi) + R_2(\eta-\xi) - R_2(1-\xi);$$
$$R_1(\zeta) = 2\{2\frac{k_2}{k} I_0(\beta_1+\beta_2,\zeta) - \frac{1}{2}\frac{(k_1+k_2)}{k}[\frac{k_2}{k_1} I_0(2\beta_2,\zeta) + I_0(2\beta_1,\zeta]\};$$
$$S_1(\zeta) = \frac{(k_1+k_2)}{k}\{I_1(\beta_1+\beta_2,\zeta) - \frac{1}{2}[I_1(2\beta_1,\zeta) + I_1(2\beta_2,\zeta)]\};$$
$$S_2(\zeta) = 2\{2\frac{k_1}{k_2} I_0(\beta_1+\beta_2,\zeta) - \frac{1}{2}\frac{(k_1+k_2)}{k}[\frac{k_1}{k_2} I_0(2\beta_2,\zeta) + I_0(2\beta_1,\zeta)]\}; \tag{10}$$
$$R_2(\zeta) = \frac{(k_1+k_2)}{k}\{I_{-1}(\beta_1+\beta_2,\zeta) - \frac{1}{2}[I_{-1}(2\beta_1,\zeta) + I_{-1}(2\beta_2,\zeta)]\};$$
$$I_0(\rho,\zeta) = \rho(\zeta^2+\rho^2)^{-1}; \qquad I_{-1}(\rho,\zeta) = -\frac{1}{2\beta}\log(\zeta^2+\rho^2);$$
$$I_1(\rho,\zeta) = \beta(\rho^2-\zeta^2)(\zeta^2+\rho^2)^{-2};$$
$$\beta = ha^{-1}; \quad \beta_i = \beta(n_i^0)^{-1/2}, i=1,2.$$

3 STRESS INTENSITY FACTOT

Similarly to classical case the stress intensity factor is determined as coefficients at a singularity in components of stress near crack periphery

$$K_I = \lim_{r\to a}[2\pi(r-a)]^{1/2} t_{33}(r,0) = -C_{44}^0 d_1^0 l_1^0 \frac{k\sqrt{\pi a}}{2k_1}\int_0^1 f(\xi)d\xi\ ,$$
$$K_{II} = \lim_{r\to a}[2\pi(r-a)]^{1/2} t_{3r}(r,0) = -C_{44}^0 (n_1^0)^{-1/2} d_1^0 \frac{k\sqrt{\pi a}}{2k_2}\beta\, g(1). \tag{11}$$

In the case of $\beta\to\infty$ can sow that $M_1\to 0, M_2\to 0, \frac{1}{\beta}N_1\to 0, \beta N_2\to 0$, and then

$$K_I^\infty \equiv \lim_{\beta\to\infty} K_I = -\frac{2}{\sqrt{\pi a}}\int_0^a \frac{t\sigma(t)dt}{\sqrt{a^2-t^2}} \quad , \quad K_{II}^\infty \equiv \lim_{\beta\to\infty} K_{II} = 0, \tag{12}$$

4. TWO PARALLEL PENNY-SHARPED CRACK

We will consider infinity body that containing two parallel penny-shaped crack of radius a which are situated in the plane $x_3=0$ and $x_3=2h$ with centre on Ox_3-axis

The initial stresses that operate along a crack correspond to biaxial uniform compression or extension is defined by

$$S_{33}^0 = 0, \qquad S_{11}^0 = S_{22}^0 \neq 0,$$
$$u_m^0 = \delta_{jm}(\lambda_j - 1)x_j; \quad \lambda_1=\lambda_2\neq\lambda_3, \quad \lambda_j = const. \tag{13}$$

The linearized problem, has the following boundary conditions on crack faces $x_3=\pm 0$ and $x_3=\pm 2h$:

$$t_{33} = \sigma(r), \quad t_{3r} = 0 \qquad (x_3 = \pm 0, \quad 0 \le r < a);$$
$$t_{33} = \sigma(r), \quad t_{3r} = 0 \qquad (x_3 = \pm 2h, \quad 0 \le r < a) \qquad (14)$$

For bend form in case $n_1^0 \neq n_2^0$ in the upper halfspace $x_3 \ge -h$:

$$t_{33} = \sigma(r), \quad t_{3r} = 0 \qquad (x_3 = \pm 0, \quad 0 \le r < a);$$
$$t_{33} = 0, \quad u_r = 0 \qquad (x_3 = -h, \quad 0 \le r < \infty); \qquad (14)$$

For this case the procedure of deriving Fredholm equations is analogous [3,4] taking into account non zero right part:

$$f(\xi) - \frac{1}{\pi k}\int_0^1 M_1(\xi,\eta) f(\eta) d\eta - \frac{2}{\pi k}\int_0^1 N_1(\xi,\eta) g(\eta) d\eta = \frac{1}{\pi k} q(\xi);$$
$$g(\xi) - \frac{1}{\pi k}\int_0^1 M_2(\xi,\eta) g(\eta) d\eta - \frac{2}{\pi k}\int_0^1 N_2(\xi,\eta) f(\eta) d\eta - C_1 = 0;$$
$$\int_0^1 g(\xi) d\xi = 0 \qquad (0 \le \xi \le 1, 0 \le \eta \le 1); \qquad (15)$$
$$q(\xi) = 4[p(0) + \xi \int_0^{\pi/2} p'(\xi \sin\zeta) d\zeta]; \quad p(\zeta) \equiv \frac{\sigma(a\zeta)}{C_{44}^0 d_1^0 l_1^0};$$
$$k = \frac{(l_1^0 - l_2^0) k_1}{k_2}.$$

5. CONCLUSIONS

One of surveyed variants for a problem with near-surface crack is presented. Problems of two parallel cracks and a system of parallel cracks surveyed also. These problems also are brought to Fredholm's integral equations of the second kind. In particular for two flaws integral equations look like (15)

Also it is shown that for a problem near-surface crack at case $\beta \to \infty$ there was conversion to a case of an isolated crack in the infinite material.

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