# NEAR-SURFACE PENNY-SHAPED CRACK AT THE INTERFACE OF PHYSICAL-MECHANICAL PROPERTIES OF A MATERIAL

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## ABSTRACT

This paper presents an analytical investigation of a nonclassical fracture mechanics axisymmetrical problem for a near-surface penny-shaped crack at the interface of physical and mechanical properties of a material. The crack is located in a near-surface layer parallel to the surface of a halfspace, which is subjected to uniform compression parallel to the crack plane. An approach proposed by Guz (1983) in the framework of the three dimensional linearised solid mechanics is used. Analysis involves reducing the problem to a system of Fredholm integral equations of the second kind, where the solutions are identified with harmonic potential functions. The representations of the stress intensity factors near the crack edges are obtained.

**Keywords**: compression, halfspace, interface crack, penny-shaped crack, stress intensity factors.

## 1. INTRODUCTION

Investigation of the failure of cracked materials under compressive loads directed along the cracks is one of the nonclassical problems of fracture mechanics. For this loading scheme, the stress-intensity factors and crack-openings are equal to zero when determined within the framework of the classical linear theory of fracture mechanics. Thus, in considering the failure of materials with such loads that do not influence the stress-intensity coefficients and the quantities characterizing the opening of the cracks, the classical fracture criteria of the Griffith-Irwin type are inapplicable. For this class of problems, therefore, an expedient fracture criterion is that proposed by Guz within the framework of three-dimensional linearized stability theory of deformable bodies. According to this criterion, the onset of fracture corresponds to local loss of stability of a material under compression along existing crack-type defect. A general approach that allows the description of such problems in a common form for different models of materials was outlined in detail in [1-3], where a complete bibliography on the subject was also given. Plane and axisymmetric problems have been investigated within this approach for some typical geometric defect distributions in various materials (highly elastic, composite, plastic) [4,5]. Similar to the problem considered in this paper, homogeneous problems for a disk-shaped surface crack – the problem of the fracture mechanics subjected to compression along cracks and the problems of the brittle fracture of solids with the initial stresses acting along the cracks surfaces – were studied by Guz and Nazarenko [6] and Kuzmin and Uflyand [7]. Solutions for initially stressed halfspace with a circular crack under normal and tangential load are presented by Babich et al. [8] and

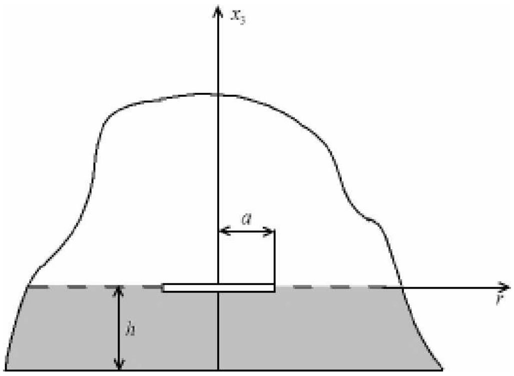
Guz et al. [9], respectively. Studies of related homogeneous problems for a penny-shaped crack near a free boundary of a semiinfinite solid (fracture mechanics of materials in compression along cracks) are presented by Guz and Nazarenko [10] and Guz etal. [11].

Analytical solutions for the stress intensity factors are available in the literature for only some bimaterial interface crack problems, see, e.g. [12-21]; they are generally for plane elasticity with relatively simple geometries and loading conditions. For the case of axisymmetry, the analytical solution of the problem of a penny-shaped crack lying at the interface between two elastic half-spaces was presented, e.g., in [22,23].

In a series of papers [24-26] A.N. Guz and I.A. Guz obtained important analytical results for the stability loss problem for the interface of two bodies. In [24] the stability loss problem for the interface of two bodies is considered in case when some plane cracks are located in the interface and bodies are compressed along the cracks (along the interface of two various materials). The investigations have been performed for the problems using the three-dimensional linearised theory of deformable bodies stability. Exact solution of the above mentioned stability loss problem was obtained for the case when the first material had unequal roots and second material had equal roots of main equation on determination of complex roots (parameters) as applied to the plane problem of three-dimensional linearised theory of deformable bodies stability. In earlier published author's articles [25,26] the exact solutions have been obtained for the cases when the first material and the second one have unequal roots or when both materials have equal roots. Some mechanical effects have been analyzed under general conditions (elastic, elasto-plastic compressible and incompressible isotropic and orthotropic bodies). It was pointed out that in accordance with the exact solutions the main result and conclusions have the same general form for the above mentioned various cases of roots.

In this article, within the above approach, we will examine a nonclassical fracture mechanics axisymmetrical problem for a near-surface penny-shaped crack at the interface of physical and mechanical properties of a material. The crack is located in a near-surface layer parallel to the surface of the halfspace, which is subjected to uniform compression parallel to the crack plane. We will limit ourselves to the case of equal roots of the characteristic equation (using the terminology in [1]).

## 2. PROBLEM FORMULATION



We will consider the stability loss problem for the interface of physical and mechanical properties of a material of a semiinfinite solid in case when penny-shaped crack is located in a near-surface layer parallel to the surface of a halfspace, which is subjected to uniform compression parallel to the crack plane (along the interface of two various materials).

## 3. APPROACH AND SOLUTION

In connection with the problem of the formation of bounded defects (cavities, laminations) at the interface of physical and mechanical properties of a material, it is important to devise mathematical methods that allow determination of the critical compressive loads corresponding to the initial stage of failure of the material. The suggested analytical approach to the solution of the given problem with the corresponding failure criteria is based on applying relations of the three-dimensional linearised solid mechanics and makes it possible to describe such problems in a standard form for different material models.

To investigate the suggested approach more deeply let us consider a model problem from the three-dimensional linearised solid mechanics. This model problem is a particular case of the general problem formulated in the beginning. The derivation of its solution, presented below, gives a qualitative picture of studied mechanical model of a near-surface penny-shaped crack at the interface of physical and mechanical properties of a material.

The crack with the radius $a$ is located in the upper halfspace $x_3 \geq -h$ in the plane $x_3 = 0$ with centre on axis $Ox_3$ (the meridional section passing through the Ox-axis is shown on the figure above). With reference to a system of Cartesian coordinates $x_j$ (j = 1, 2, 3), the components of the displacement vector are given by $u_j$ and the components of the Kirchhoff stress tensor by $t_{ij}$.

The boundary conditions of the linearized problem (edges of the crack and the boundary of the half-space are not loaded) have the form

$$\begin{aligned} &t_{3r} = 0, \quad t_{33} = 0, \quad \left(x_3 = \pm 0, \quad 0 \leq r < a\right); \\ &t_{3r} = 0, \quad t_{33} = 0, \quad \left(x_3 = -h, \quad 0 \leq r < \infty\right); \end{aligned} \qquad (1)$$

The linearized equilibrium equation in the displacements $u_j$ for incompressible solids has the form

$$\begin{aligned} &\kappa_{im\alpha\beta} \frac{\partial^2 u_\alpha}{\partial x_i \partial x_\beta} + q_{\alpha m} \frac{\partial p}{\partial x_\alpha} = 0, \\ &q_{m\alpha} \frac{\partial u_\alpha}{\partial x_m} = 0, \quad (i, m, \alpha, \beta = 1,2,3), \end{aligned} \qquad (2)$$

The linearized relations for incompressible bodies have the form

$$\begin{aligned} &t_{ij} = \kappa_{ij\alpha\beta} \frac{\partial u_\alpha}{\partial x_\beta} + q_{ij} p, \quad q_{mk} = \delta_{mk} q_m, \\ &\kappa_{ij\alpha\beta} = \lambda_j \lambda_\alpha \left[ \delta_{ij} \delta_{\alpha\beta} a_{i\beta} + \left(1 - \delta_{ij}\right)\left(\delta_{i\alpha}\delta_{j\beta} + \delta_{i\beta}\delta_{j\alpha}\right)\mu_{ij} \right] + \delta_{i\beta}\delta_{j\alpha} S^0_{\beta\beta}. \end{aligned} \qquad (3)$$

Here the quantities $q_{mk}$, $\alpha_{i\beta}$, $\mu_{ij}$ are determined as functions of the chosen potential.

The general solution of the linearized equations for equal roots in axialsymmetric case are representable in the form [1]

$$\begin{aligned}
u_r &= -\frac{\partial \phi}{\partial r} - z_1 \frac{\partial F}{\partial r}, \\
u_3 &= \left(n_1^0\right)^{-1/2} \left(m_1^0 - m_2^0 - 1\right) F - m_1^0 \Phi - m_1^0 z_1 \frac{\partial F}{\partial z_1}, \\
t_{33} &= C_{44}^0 \left[ \left(d_1^0 l_1^0 - d_2^0 l_2^0\right) \frac{\partial F}{\partial z_1} - d_1^0 l_1^0 \frac{\partial \Phi}{\partial z_1} - d_1^0 l_1^0 z_1 \frac{\partial^2 F}{\partial z_1^2} \right], \\
t_{3r} &= C_{44}^0 \frac{\partial}{\partial r} \left[ \left(d_1^0 - d_2^0\right) F - d_1^0 \Phi - d_1^0 z_1 \frac{\partial F}{\partial z_1} \right].
\end{aligned} \tag{4}$$

Where $\varphi(r, z_1)$, $F(r, z_1)$ - potential harmonic functions, $\Phi \equiv \partial\varphi/\partial z_1$, $z_1 = \left(n_1^0\right)^{-1/2} x_1$.

The investigations were carried out using Henkel's integral transformations along the radial coordinate r (transformation of the zero order), for the problem reduced to a system of paired integral equations (by the substitution method). Finally, using the method proposed by Ya. S. Uflyand [28] a system of Fredholm's integral equations of the second kind with an additional condition was obtained (using the solution of Schlemilch's integral equation). The resulting system of the integral equations has the form

$$\begin{gathered}
f(\xi) + \frac{1}{\pi k}\int_0^1 M_1(\xi,\eta) f(\eta)\, d\eta - \frac{2}{\pi k}\int_0^1 N_1(\xi,\eta) g(\eta)\, d\eta = \frac{1}{\pi k} q(\xi); \\
g(\xi) + \frac{1}{\pi k}\int_0^1 M_2(\xi,\eta) g(\eta)\, d\eta - \frac{2}{\pi k}\int_0^1 N_2(\xi,\eta) f(\eta)\, d\eta - \text{const} = 0; \\
\int_0^1 g(\xi)\, d\xi = 0, \quad 0 \le \xi \le 1, \quad 0 \le \eta \le 1;
\end{gathered} \tag{5}$$

$$\begin{gathered}
M_1(\xi,\eta) = R_1(\eta+\xi) + R_1(\eta-\xi) - R_1(1+\xi) - R_1(1-\xi); \\
N_1(\xi,\eta) = S_1(\eta+\xi) + S_1(\eta-\xi); \quad M_2(\xi,\eta) = S_2(\eta+\xi) + S_2(\eta-\xi); \\
N_2(\xi,\eta) = R_2(\eta+\xi) + R_2(\eta-\xi) - R_2(1+\xi) - R_2(1-\xi); \\
R_1(x) = -2\left[\frac{k}{2} L_0(x) + L_1(x) + \frac{1}{k} L_2(x)\right] \quad \text{and so on}; \\
L_0(x) = 2\left(n_1^0\right)^{-1/2} \beta b^{-1} \quad \text{and so on}; \quad \beta = ha^{-1}, \quad b = x^2 + 4\left(n_1^0\right)^{-1} \beta^2 .
\end{gathered} \tag{6}$$

Presented results are developed in more details in [6].

Proceeding as in the classical case [30], we determine the stress intensity factors as coefficients with singularities in the stress components near the edge of the crack

$$\begin{aligned}
K_I &= \lim_{r \to +a} \left[2\pi (r-a)\right]^{1/2} t_{33}(r,0), \\
K_{II} &= \lim_{r \to +a} \left[2\pi (r-a)\right]^{1/2} t_{3r}(r,0).
\end{aligned} \tag{7}$$

Examining $t_{33}(r,0)$ and $t_{3r}(r,0)$ we find that for $r \to +a$

$$\frac{K_I}{\sqrt{a}} = \gamma_I \frac{k}{2k_1} \int_0^1 f(\xi) d\xi,$$
$$\frac{K_{II}}{\sqrt{a}} = \gamma_{II} \frac{k}{2k_2} \beta\, g(1). \qquad (8)$$

It follows from (8) that the presence of the free surface leads to a nontrivial stress intensity factor $K_{II}$. With a limitless increase in the distance $h$ from the crack to the free boundary, we obtain the case of a crack in an infinite material (in general formulation we obtain the case of a penny-shaped crack at the interface of two halfspaces).

It can be shown that a reduction of the dimensionless distance between the crack and the free surface of the material $\beta = ha^{-1}$ results in a large reduction of the critical stresses in comparison with the loads for a separate crack in an infinite material (crack at the interface of two halfspaces). When $\beta \to \infty$ the critical loads approach asymptotically the values of the loads corresponding to the axisymmetric problem for a separate crack in an infinite material (crack at the interface of two halfspaces).

## 4. CONCLUSIONS

The stability loss problem for the interface of physical and mechanical properties of a material of a semiinfinite solid in case when penny-shaped crack is located in a near-surface layer parallel to the surface of a halfspace, which is subjected to uniform compression parallel to the crack plane (along the interface of two various materials) has been considered in this paper based on the approach proposed by Guz (1983) within the scope of the linearized mechanics of deformable solids. Analysis involves reducing the problem to a system of Fredholm integral equations of the second kind, where the solutions are identified with harmonic potential functions. The representations of the stress intensity factors near the crack edges are obtained. The obtained results lead to the following conclusions:

Similar to the classical case the presence of a free boundary in a solid leads to a nontrivial stress intensity factor $K_{II}$. As the relative distance between the crack and the halfspace boundary tends to infinity, the stress intensity factors tend to the values corresponding the case of a penny-shaped crack at the interface of two halfspaces.

The distance between the crack and the free surface results in a large reduction of the critical stresses in comparison with the loads for a separate crack at the interface of two halfspaces. As distance between the crack and the free surface tends to infinity, the critical loads approach asymptotically the values of the loads corresponding to the axisymmetric problem for a penny-shaped crack at the interface of two halfspaces.

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