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THE CRITICAL VELOCITY OF A MOVING TIME-HORMONIC LOAD ACTING ON A PRE-STRESSED PLATE RESTING ON A PRE-STRESSED HALF-PLANE

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ABSTRACT

Within the framework of the piecewise homogeneous body model with the use of the Three-dimensional Linearized Theory of Elastic Waves in Initially Stressed Bodies the investigations are carried out on the dynamical behavior of a half-plane covered by the pre-stretched layer under the action of the moving linearly located time-harmonic forces. Two types (complete and incomplete) of contact conditions on the influence are considered. The subsonic state is considered, and numerical results for the critical speed of the moving load and for the illustrating the influence of the load frequency of this speed are presented. According to these results, in particular, it is established that the interaction between the moving velocity and vibrating frequency of the external load change significantly the values of the critical speed (velocity). Concrete numerical results are presented for the pair of materials Steel (layer) + Aluminium (half-plane).

Keywords: Moving Time-Harmonic Forces, Initial stress, Critical Velocity, Half-plane, Covering Layer.

1. INTRODUCTION

Under acting a moving load on the layered half-space (or half plane) for the certain values of the velocity of this load the resonance type phenomenon takes place. As usually, these values of the velocity is called as ‘’critical velocity’’. Consequently, the critical velocity of the moving load can be taken as one of the dynamical failure parameters of the layered materials. Therefore the investigations of the problems regarding the dynamical response of the layered materials to a moving load has a great significance for estimation of the dynamical load carried or failure capacity of the layered systems.

Up to now a lot of numbers of investigations were made in this field. We here consider a brief reviewer some of them which regard the study made in the present paper. We begin this reviewer with papers [1-3] in which the dynamical response of the systems consisting of a packet of layers and a half-space to the moving load action were studied. In these investigations the motion of layers were described by the Kirchoff or Timeshenko beam theory, but the motion of the half-space by the linear theory of elastic waves.

It should be noted that many modern elastodynamics problems wit moving loads cannot be solved within the framework of the classical linear theory of elastic waves among which are also those for initially stressed layered bodies. Initial stresses occur in layered systems after their manufacture and assembly, and they must be taken into account in studying the dynamical response of such systems to a moving load. Within the scope of certain conditions the foregoing type of problems can be solved with the use of the Tree-dimensional Linearized Theory of Elastic Waves in Initially Stressed Bodies (TLTEWISB). The construction of field equations of this theory and its application for investigation of the wave propagation problems for initially stressed bodies are detailed in [14].

In [5-7], time-harmonic stress fields in layered pre-stressed bodies were examined by employing the TLTEWISB. At the some time, only a few studies have been performed until now on the dynamical response of a pre-stressed layered half-space to a moving load [8-10]. In [8], the dynamical response of a system consisting of a layer and pre-stressed half-plane was considered. The motion of the covering layer was described by the Timoshenko beam theory, but the motion of the half-plane by the TLTEWISB. The solution to the corresponding boundary-value problem was found by employing Fourier transforms. Numerical investigations were performed in the case where the constitutive relations for the half-plane material were given by a harmonic potential. As a result, the influence of problem parameters on the critical velocity was clarified. In [9], the problem considered in [8] was studied by using the complex potentials of the TLTEWISB. In the paper [10], the investigations carried out in [8, 9] are developed for the case where the covering layer has also an initial stress, and the equation of motion of this layer is given by use of the TLTEWISB. The influence of the problem parameters on the critical speed of the moving load is studied.

Among the steady-state problems with moving forces the problems with moving and oscillating forces are the most complicated ones. Studies on the dynamical response of layered bodies to moving and simultaneously oscillating loads are of great significance in both theoretical and practical sense. Nowadays such problems were investigated only for the homogeneous isotrop and anisotrop materials [11-14]. In the present paper the attempt is made for development of such investigation for the pre-stressed stratified half-plane. This attempt is made within the framework of the piecewise homogeneous body model by the use of the TLTEWISB. The investigations carried out in the present paper can be also considered as the development of the investigations [7, 10] for the moving and simultaneously oscillating loads. As a result of the present investigation, in particular, it is determined the influence of the oscillating frequency on the values of the critical velocity of the moving load.

2. PROBLEM FORMULATION

Let us consider the pre-stressed half-plane covered by pre-stressed layer. The points of the layer and half-plane are specified by the Lagrangian coordinates in a Cartesian reference frame $Ox_1x_2x_3$ (Fig. 1). Before assembling, the layer and the half-plane had been stressed separately in the Ox_1 -axis direction, so that a uniaxial homogeneous initial stress state exists in each of them. The covering layer and the half-plane occupy the regions $\{-\infty < x_1 < \infty$,

$-h \le x_2 \le 0$, $-\infty < x_3 < +\infty\}$, $\{-\infty < x_1 < +\infty$, $-\infty \le x_2 \le -h$, $-\infty < x_3 < \infty\}$, respectively (Fig.1). The Ox_3 axis is perpendicular to the figure plane and therefore is not shown in Fig. 1.

The quantities related to the related to the layer and half-plane are denoted by the superscripts (1) and (2), respectively, but the quantities related to the initial stresses are denoted by the superscripts (m)0, where m=1,2.

The linearly elastic materials of the layer and half-plane are taken to be homogeneous, isotropic, and moderately rigid. The initial stress state in the constituents can be written in the form

$$\sigma_{11}^{(m)0} = const\,,\ \sigma_{ij}^{(m)0} = 0 \ \text{ at } \ ij \neq 11 \ \text{(m=1,2).}$$

Thus, within the framework of the foregoing assumptions, we assume that, on the free upper face of the covering layer, a linearly located time-harmonic force moving with constant velocity V in the direction of the Ox_1 axis acts (Fig. 1). It is required to determine the dynamical response of this layered system to the foregoing loading.

We will consider only the subsonic state, i.e.,

$$V < \min(C_2^{(1)};\ C_2^{(2)}),\ C_2^{(m)} = \sqrt{\frac{\mu^{(m)}}{\rho^{(m)}}},\ m = 1,2$$

where $C^{(m)}, \mu^{(m)}$ and $\rho^{(m)}$ are the speed of distortion waves, the shear modulus and density, respectively, of constituent materials.

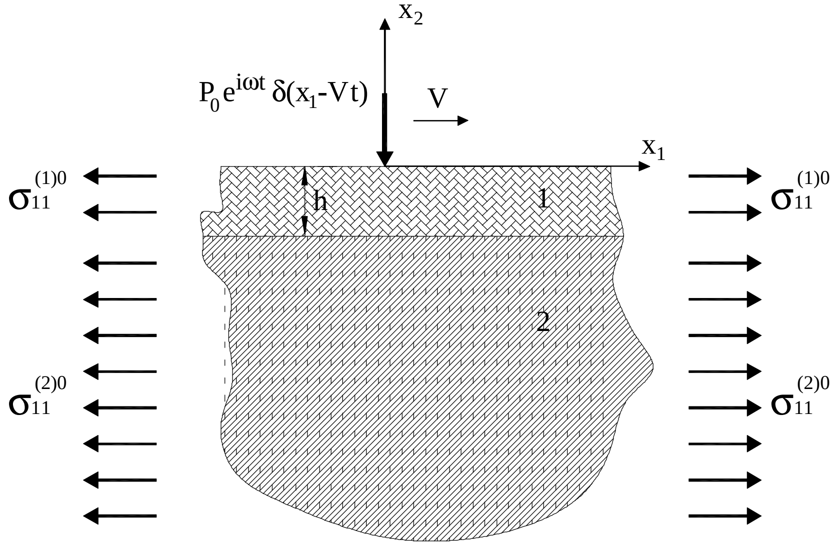

Fig. 1. A geometry of considered system

According to [4], for the considered plane-strain state the complete system of equations of motion of the TLTEWISB in the case of small initial strains is

$$\begin{aligned} &\frac{\partial\sigma_{11}^{(m)}}{\partial x_1}+\frac{\partial\sigma_{21}^{(m)}}{\partial x_2}+\sigma_{11}^{(m),0}\frac{\partial^2 u_1^{(m)}}{\partial x_1^2}=\rho^{(m)}\frac{\partial^2 u_1^{(m)}}{\partial t^2} \\ &\frac{\partial\sigma_{12}^{(m)}}{\partial x_1}+\frac{\partial\sigma_{22}^{(m)}}{\partial x_2}+\sigma_{11}^{(m),0}\frac{\partial^2 u_2^{(m)}}{\partial x_1^2}=\rho^{(m)}\frac{\partial^2 u_2^{(m)}}{\partial t^2} \end{aligned} \tag{1}$$

$$\sigma_{ij}^{(m)}=\lambda^{(m)}\theta^{(m)}\delta_i^j+2\mu^{(m)}\varepsilon_{ij}^{(m)} \tag{2}$$

$$\theta^{(m)}=\varepsilon_{11}^{(m)}+\varepsilon_{22}^{(m)},\ \varepsilon_{ij}^{(m)}=\frac{1}{2}\left(\frac{\partial u_i^{(m)}}{\partial x_j}+\frac{\partial u_j^{(m)}}{\partial x_i}\right) \tag{3}$$

where i, j, m=1,2; $\lambda^{(m)}$ and $\mu^{(m)}$ are Lame' constants. The other notation used in (1)-(3) is conventional.

Note that Eqs. (1)-(3) are satisfied within the covering layer and the half-plane separately. According to the above-mentioned, on the upper face of covering layer, the following boundary conditions must be satisfied

$$\sigma_{12}^{(1)}\Big|_{x_2=0}=0,\ \sigma_{22}^{(1)}\Big|_{x_2=0}=Pe^{i\omega t}\delta(x_1-Vt) \tag{4}$$

where $\delta(x)$ is the Dirac function.

As in [8, 10], we will consider two types (complete and incomplete) of contact conditions on the interface plane between the covering layer and half-plane. For the complete conditions, we assume

$$\sigma_{i2}^{(1)}\Big|_{x_2=-h}=\sigma_{i2}^{(2)}\Big|_{x_2=-h},\ u_i^{(1)}\Big|_{x_2=-h}=u_i^{(2)}\Big|_{x_2=-h},\ \text{i=1,2} \tag{5}$$

and for the incomplete ones

$$\sigma_{12}^{(1)}\Big|_{x_2=-h}=0,\ \sigma_{12}^{(2)}\Big|_{x_2=-h}=0,\ \sigma_{22}^{(1)}\Big|_{x_2=-h}=\sigma_{22}^{(2)}\Big|_{x_2=-h},\ u_2^{(1)}\Big|_{x_2=-h}=u_2^{(2)}\Big|_{x_2=-h} \tag{6}$$

In addition, we also assume that

$$\left|u_i^{(2)}\right|,\left|\sigma_{ij}^{(2)}\right|<M=\text{constant as } x_2\to-\infty\,. \tag{7}$$

This terminates the formulation of the problem. In the case where initial stresses are absent in the layer and half-plane the described formulation coincide with the corresponding one of the classical linear theory of the elastodynamics.

Thus, the determination of the dynamical failure parameter, i.e. the critical velocity for the moving time-harmonic load is reduced to the solution of the boundary-value and contact problem (1)-(7).

3. SOLUTION METHOD

From equations (1)-(3) we obtain the following equations of the motion in terms of perturbations of displacements u_i:

$$\begin{aligned} &\left(\lambda^{(m)}+2\mu^{(m)}+\sigma_{11}^{(m),0}\right)\frac{\partial^2 u_1^{(m)}}{\partial x_1^2}+\mu^{(m)}\frac{\partial^2 u_1^{(m)}}{\partial x_2^2}+\left(\lambda^{(m)}+\mu^{(m)}\right)\left(\frac{\partial^2 u_2^{(m)}}{\partial x_1\partial x_2}\right)=\rho^{(m)}\frac{\partial^2 u_1^{(m)}}{\partial t^2} \\ &\left(\lambda^{(m)}+\mu^{(m)}\right)\left(\frac{\partial^2 u_1^{(m)}}{\partial x_1\partial x_2}\right)+\left(\mu^{(m)}+\sigma_{11}^{(m),0}\right)\frac{\partial^2 u_2^{(m)}}{\partial x_1^2}+\left(\lambda^{(m)}+2\mu^{(m)}\right)\frac{\partial^2 u_2^{(m)}}{\partial x_2^2}=\rho^{(m)}\frac{\partial^2 u_2^{(m)}}{\partial t^2} \end{aligned} \tag{8}$$

in the coordinate system

$$x_2' = x_2 \ ve \ x_1' = x_1 - V\tau \tag{9}$$

which moves together with the loading force, Eqs. (8) have the form

$$\begin{aligned}
&\left(\lambda^{(m)} + 2\mu^{(m)} + \sigma_{11}^{(m)0}\right)\frac{\partial^2 u_1^{(m)}}{\partial x_1^2} + \mu^{(m)}\frac{\partial^2 u_1^{(m)}}{\partial x_2^2} + \left(\lambda^{(m)} + \mu^{(m)}\right)\left(\frac{\partial^2 u_2^{(m)}}{\partial x_1 \partial x_2}\right) = \\
&\rho^{(m)}\left(\frac{\partial^2 u_1^{(m)}}{\partial t^2} - 2V\frac{\partial^2 u_1^{(m)}}{\partial x_1 \partial t} + V^2\frac{\partial^2 u_1^{(m)}}{\partial {x_1}^2}\right) \\
&\left(\lambda^{(m)} + \mu^{(m)}\right)\left(\frac{\partial^2 u_1^{(m)}}{\partial x_1 \partial x_2}\right) + \left(\mu^{(m)} + \sigma_{11}^{(m)0}\right)\frac{\partial^2 u_2^{(m)}}{\partial x_1^2} + \left(\lambda^{(m)} + 2\mu^{(m)}\right)\frac{\partial^2 u_2^{(m)}}{\partial x_2^2} = \\
&\rho^{(m)}\left(\frac{\partial^2 u_2^{(m)}}{\partial t^2} - 2V\frac{\partial^2 u_2^{(m)}}{\partial x_1 \partial t} + V^2\frac{\partial^2 u_2^{(m)}}{\partial {x_1}^2}\right)
\end{aligned} \tag{10}$$

where the primes of x_1 and x_2 are omitted. In this case the second boundary condition in (4) is replaced by

$$\sigma_{22}^{(1)}\Big|_{x_2=0} = Pe^{i\omega t}\delta(x_1) \tag{11}$$

According to (11), the sought values can be presented as $g(x_1, x_2, t) = \overline{g}(x_1, x_2)e^{i\omega\tau}$. After this presentation, we obtain the following equations for the amplitude of displacement $\overline{u}_1$ and $\overline{u}_2$, from equations (10).

$$\begin{aligned}
&\left(2 + \frac{\lambda^{(m)}}{\mu^{(m)}} + \frac{\sigma_{11}^{(m)0}}{\mu^{(m)}}\right)\frac{\partial^2 \overline{u}_1^{(m)}}{\partial x_1^2} + \frac{\partial^2 \overline{u}_1^{(m)}}{\partial x_2^2} + \left(1 + \frac{\lambda^{(m)}}{\mu^{(m)}}\right)\left(\frac{\partial^2 \overline{u}_2^{(m)}}{\partial x_1 \partial x_2}\right) = \\
&\left(-\frac{\omega^2}{{C_2^{(m)}}^2}\overline{u}_1^{(m)} - i\frac{2V\omega}{{C_2^{(m)}}^2}\frac{\partial \overline{u}_1^{(m)}}{\partial x_1} + \frac{V^2}{{C_2^{(m)}}^2}\frac{\partial^2 \overline{u}_1^{(m)}}{\partial {x_1}^2}\right) \\
&\left(1 + \frac{\lambda^{(m)}}{\mu^{(m)}}\right)\left(\frac{\partial^2 \overline{u}_1^{(m)}}{\partial x_1 \partial x_2}\right) + \left(1 + \frac{\sigma_{11}^{(m)0}}{\mu^{(m)}}\right)\frac{\partial^2 \overline{u}_2^{(m)}}{\partial x_1^2} + \left(2 + \frac{\lambda^{(m)}}{\mu^{(m)}}\right)\frac{\partial^2 \overline{u}_2^{(m)}}{\partial x_2^2} = \\
&\left(-\frac{\omega^2}{{C_2^{(m)}}^2}\overline{u}_2^{(m)} - i\frac{2V\omega}{{C_2^{(m)}}^2}\frac{\partial \overline{u}_2^{(m)}}{\partial x_1} + \frac{V^2}{{C_2^{(m)}}^2}\frac{\partial^2 \overline{u}_2^{(m)}}{\partial {x_1}^2}\right)
\end{aligned} \tag{12}$$

The boundary condition (10) is replaced by the following one

$$\overline{\sigma}_{22}^{(1)}\Big|_{x_2=0} = P\delta(x_1). \tag{13}$$

The other conditions in (4)-(7) remain valid in coordinates (9). In order to solve equation system (12), we apply the Fourier transform

$$f_F(s,x_2)=\int_{-\infty}^{+\infty} f(x_1,x_2)e^{-isx_1}dx_1$$

to it. Below we use the dimensionless coordinates $\tilde{x}_1 = x_1/h,\ \tilde{x}_2 = x_2/h$ As a result, we have

$$\begin{gathered}
-s^2a^{(m)}\overline{u}_{1_F}^{(m)}+isb^{(m)}\frac{d\overline{u}_2^{(m)}}{d\tilde{x}_2}+\frac{d^2\overline{u}_{1_F}^{(m)}}{d\tilde{x}_2^2}=0 \\
-s^2c^{(m)}\overline{u}_{2_F}^{(m)}+isb^{(m)}\frac{d\overline{u}_{1_F}^{(m)}}{d\tilde{x}_2}+d^{(m)}\frac{d^2\overline{u}_{2_F}^{(m)}}{d\tilde{x}_2^2}=0
\end{gathered} \tag{14}$$

where

$$\begin{gathered}
a^{(m)}=2+\frac{\lambda^{(m)}}{\mu^{(m)}}+\frac{\sigma_{11}^{(m)0}}{\mu^{(m)}}-\frac{\omega^2h^2}{s^2C_2^{(m)^2}}+\frac{2V\omega h}{sC_2^{(m)^2}}-\frac{V^2}{C_2^{(m)^2}},\quad b^{(m)}=1+\frac{\lambda^{(m)}}{\mu^{(m)}} \\
c^{(m)}=1+\frac{\sigma_{11}^{(m)0}}{\mu^{(m)}}-\frac{\omega^2h^2}{s^2C_2^{(m)^2}}+\frac{2V\omega h}{sC_2^{(m)^2}}-\frac{V^2}{C_2^{(m)^2}},\ d^{(m)}=2+\frac{\lambda^{(m)}}{\mu^{(m)}},\ i=\sqrt{-1}
\end{gathered} \tag{15}$$

After some mathematical manipulations, we obtain from (14) the following equation:

$$\begin{gathered}
(b^{(m)}+1)\frac{d^4\overline{u}_{2_F}^{(m)}}{d\tilde{x}_2^4}+\left[s^2(b^{(m)})^2-s^2c^{(m)}-s^2a^{(m)}(b^{(m)}+1)\right]\frac{d^2\overline{u}_{2_F}^{(m)}}{d\tilde{x}_2^2}+s^4a^{(m)}c^{(m)}\overline{u}_{2_F}^{(m)}=0 \\
\frac{d\overline{u}_{1_F}^{(m)}}{d\tilde{x}_2}=-is\frac{c^{(m)}}{b^{(m)}}\overline{u}_{2_F}^{(m)}+\frac{i(b^{(m)}+1)}{sb^{(m)}}\frac{d^2\overline{u}_{2_F}^{(m)}}{d\tilde{x}_2^2}
\end{gathered} \tag{16}$$

From equation (16) it follows

$$\begin{gathered}
\overline{u}_{2_F}^{(1)}=F_1^{(1)}(s)e^{K_1^{(1)}x_2}+F_2^{(1)}(s)e^{-K_1^{(1)}x_2}+F_3^{(1)}(s)e^{K_2^{(1)}x_2}+F_4^{(1)}(s)e^{-K_2^{(1)}x_2} \\
\overline{u}_{2_F}^{(2)}=F_1^{(2)}(s)e^{K_1^{(2)}x_2}+F_3^{(2)}(s)e^{K_2^{(2)}x_2}
\end{gathered} \tag{17}$$

where

$$\begin{gathered}
K_1^{(m)}=\sqrt{-\frac{A^{(m)}}{2}+\sqrt{\left(\frac{A^{(m)}}{2}\right)^2-B^{(m)}}}\ ,\ K_2^{(m)}=\sqrt{-\frac{A^{(m)}}{2}-\sqrt{\left(\frac{A^{(m)}}{2}\right)^2-B^{(m)}}} \\
A^{(m)}=\frac{-s^2a^{(m)}(b^{(m)}+1)+s^2b^{(m)^2}-s^2c^{(m)}}{b^{(m)}+1},B^{(m)}=\frac{s^4a^{(m)}c^{(m)}}{b^{(m)}+1}
\end{gathered} \tag{18}$$

Thus, from eqs. (17), (16), (2) and (3), the Fourier transforms of all sought quantities are found. The unknowns $F_1^{(1)}(s),...,F_4^{(1)}(s),F_1^{(2)}(s)$ and $F_3^{(2)}(s)$ appearing in these transforms are obtained from boundary condition (13) and contact condition (5) (for a complete contact) or (6) (for an incomplete contact), which form a closed system of algebra equations. Note that condition (7) was already used under determining $u_{2F}^{(m)}$ in (17).

From the algebraic equations, we find the aforementioned unknowns and then employ the inverse transform

$$f(x_1,x_2)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} f_F(s,x_2)e^{isx_1}ds \tag{19}$$

As follows from the equations (15), integral (19) cannot be presented as $\frac{1}{2\pi}\int_{-\infty}^{+\infty}(.)e^{isx_1}ds=\frac{1}{\pi}\int_{0}^{+\infty}(.)\cos(sx_1)ds$ or $\frac{1}{2\pi}\int_{-\infty}^{+\infty}(.)e^{isx_1}ds=\frac{1}{\pi}\int_{0}^{+\infty}(.)\sin(sx_1)ds$. Because, the expression of $a^{(m)}$ and $c^{(m)}$ in (15) contains the term $2V\omega h/\left(sC_2^{(m)^2}\right)$ which is not even one with respect to s. Note that such term is not appear under action moving load without oscillation [10] or under action of the time-harmonic load without moving [7] on the considered stratified half-plane. Therefore, in the present investigation the calculation of the integral (19) is more complicate than that in the investigations [7, 10]. In connection with this, in the present paper the algorithm proposed in [7, 10] for calculation of the integral (19) is developed for the considered case. Under this development the improper integral (19) is replaced by the corresponding proper integrals by using the relation

$$\int_{-\infty}^{+\infty}(.)ds=\int_{-S_*}^{+S_*}(.)ds \tag{20}$$

The values of S_* in (20) are determined from convergence criteria for the improper integrals.

Numerical investigations show that, in general, at fixed values of problem parameters, for any value of speed V, the function $\bar{u}_{iF}^{(m)}$ and $\bar{\sigma}_{ijF}^{(m)}$ have singular points with respect to sh. Let us determine these points. The unknowns $F_1^{(1)}(s),...,F_4^{(1)}(s),F_1^{(2)}(s)$ and $F_3^{(2)}(s)$ in (17) can be expressed from the aforementioned system of algebraic equations in the form

$$F_1^{(1)}(s),...,F_4^{(1)}(s),F_1^{(2)}(s),F_3^{(2)}(s)=\frac{1}{\det\left\|\alpha_{ij}(s)\right\|}\Big(\det\left\|\beta_{ij}^{F_1^{(1)}(s)}\right\|,...,\det\left\|\beta_{ij}^{F_4^{(1)}(s)}\right\|,$$
$$\det\left\|\beta_{ij}^{F_1^{(2)}(s)}\right\|,...,\det\left\|\beta_{ij}^{F_3^{(2)}(s)}\right\|\Big)$$

The singular points coincide with roots of the equation in *s*

$$\det\left\|\alpha_{ij}(s)\right\|=0,\ i;\ j=1,\ 2,...,\ 6 \tag{21}$$

where $\alpha_{ij}(s)$ are coefficients of unknowns in the system. Note that the expressions for $\det\left\|\beta_{ij}^{F_1^{(1)}(s)}\right\|,...,\det\left\|\beta_{ij}^{F_4^{(1)}(s)}\right\|,\det\left\|\beta_{ij}^{F_1^{(2)}(s)}\right\|,...,\det\left\|\beta_{ij}^{F_3^{(2)}(s)}\right\|$ are obtained from $\left\|\alpha_{ij}(s)\right\|$ by replacing the corresponding column of $\left\|\alpha_{ij}(s)\right\|$ with the right side of the equation system.

The order of singularity (denoted by r) of the integrated values coincide with the order of roots Eq. (21). It is known that, it $0\le r<1$, integral (20) can be calculated by using a well-known algorithm. For $r=1$, the calculation of the integral is performed in the sense of Couchy principal value. In the case $r>1$, the integral does not have any meaning, and the speed corresponding to this case is called the ‘’critical velocity’’. As it was noted in the introduction of paper, at the critical speed, a resonance-type phenomenon takes place.

It is obvious that the critical speed corresponds to the local minimum (or maximum) of the function $V=V(sh)$ which satisfies Eq. (21). The main problem for layered materials under a moving and singultoniously oscillating load in the subsonic state is the determination of the critical velocity (denoted by $\upsilon_{cr.}$) and investigation of the influence oscillation frequency on the values of υ_{cr}. Now, we will consider the numerical results regarding the υ_{cr}.

4. NUMERİCAL RESULTS AND DISCUSSION

The following materials are selected for the considerations: Aluminum (half-plane) with properties $E^{(2)} = 70056900$ kN/m^2, $\nu^{(2)} = 0,35$, $\rho^{(2)} = 2,7$ kg/m^3, Steel (covering layer) with properties $E^{(1)} = 208954783$ kN/m^2, $\nu^{(1)} = 0,29$, $\rho^{(1)} = 7,86$ kg/m^3.

The validity of the algorithm and programs by the use of which the numerical investigations are carried out, is proven by the following statements:

- for the case where $V = 0$ $(\omega = 0)$ obtained numerical results coincide with the corresponding ones obtained in the paper [7] ([10]).

Introduce the following notation

$$\upsilon = \frac{V}{C_2^{(1)}},\ \Omega^2 = \frac{\omega^2 h^2}{C_2^{(1)^2}},\ \eta_1 = \frac{\sigma_{11}^{(1)0}}{E^{(1)}}, \eta_2 = \frac{\sigma_{11}^{(2)0}}{E^{(2)}}$$

Fig. 2 shows the graphs of the dependencies between υ and sh for various values of Ω under complete contact condition (5) for $\eta_1 = 0.01, \eta_2 = 0.00$. Note that in this figure the parts of the graphs which correspond the case where $sh \geq 0$ are given only. Because, it was not observed any local extremum of those under $sh < 0$. It follows from these graphs that the values of υ_{cr} increase with Ω.

Consider Table 1 which shows the values of $\upsilon_{cr.}$ for complete (upper number) (5) and for incomplete (lower number) (6) conditions for various Ω, η_1 and η_2. In this table the case for which there is not $\upsilon_{cr.}$ is indicated by dash.

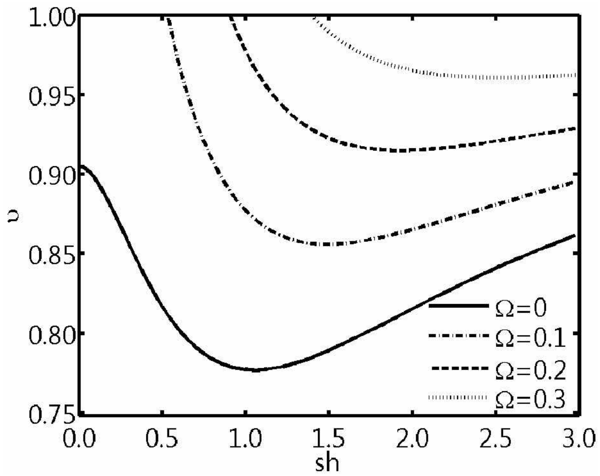

Fig. 2. The graphs of the dependencies between υ and sh for various values of Ω under complete contact conditions for the case where $\eta_1 = 0.01, \eta_2 = 0.00$.

Table 1. The values of υ_{cr} for complete (upper numbers) and incomplete (lower numbers) contact conditions for Ω, η_1, η_2

υ_{cr}		Contact	$\Omega=0$	$\Omega=0.10$	$\Omega=0.20$	$\Omega=0.30$	$\Omega=0.40$	$\Omega=0.50$	$\Omega=0.60$
$\eta_2=0$	$\eta_1=0.000$	complete	0.7644	0.8423	0.901	0.9467	-	-	-
		incomplete	0.6583	0.7413	0.8093	0.8667	0.9153	0.9556	-
	$\eta_1=0.005$	complete	0.7707	0.8491	0.908	0.9537	-	-	-
		incomplete	0.6651	0.7486	0.8167	0.8742	0.9226	0.9627	-
	$\eta_1=0.010$	complete	0.7768	0.8558	0.9149	0.9606	-	-	-
		incomplete	0.6718	0.7558	0.8240	0.8815	0.9298	0.9697	-
	$\eta_1=0.030$	complete	0.8005	0.8819	0.9417	0.9875	-	-	-
		incomplete	0.6977	0.7835	0.8523	0.9097	0.9576	0.9966	-
$\eta_1=0$	$\eta_2=0.005$	complete	0.7665	0.8438	0.9021	0.9474	-	-	-
		incomplete	0.6610	0.7435	0.8112	0.8684	0.9167	0.9567	-
	$\eta_2=0.010$	complete	0.7686	0.8452	0.9032	0.9482	-	-	-
		incomplete	0.6638	0.7457	0.8131	0.8699	0.9180	0.9579	-
	$\eta_2=0.030$	complete	0.7764	0.8507	0.9072	0.951	-	-	-
		incomplete	0.6743	0.7542	0.8201	0.8759	0.9231	0.9620	-
	$\eta_2=-0.005$	complete	0.7623	0.8408	0.9000	0.9459	-	-	-
		incomplete	0.6554	0.7390	0.8074	0.8651	0.9140	0.9544	-
	$\eta_2=-0.010$	complete	0.7601	0.8393	0.8988	0.9451	-	-	-
		incomplete	0.6525	0.7366	0.8054	0.8634	0.9125	0.9532	-
	$\eta_2=-0.030$	complete	0.7509	0.8330	0.8942	0.9417	-	-	-
		incomplete	0.6402	0.7267	0.7971	0.8563	0.9064	0.9480	-

5. CONCLUSIONS

According to data given in Table 1, we can make the following conclusions:

– the oscillations of the moving load causes to increase of the values of $\upsilon_{cr.}$, so that, the values of $\upsilon_{cr.}$ increase monotonically with Ω;

– the values of $\upsilon_{cr.}$ obtained for complete contact conditions (5) are greater than corresponding ones obtained for incomplete contact conditions (6);

– as a result of pre-stretching of the covering layer and of the half-plane the values of $\upsilon_{cr.}$ increase;

– the existence of the initial compression in the half-plane causes to decrease of values of $\upsilon_{cr.}$

Although the above-written conclusions were attained for the concrete selected pair of materials, in the qualitative sense they have also a general meaning.

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